***Statistical methods applied in microelectronics***

Dipartimento di Scienze Statistiche
Università Cattolica del Sacro Cuore
Milan, 13/6/2011

UNIVERSITÀ CATTOLICA del Sacro Cuore

# Multivariate Statistical Process Control: *an introduction*

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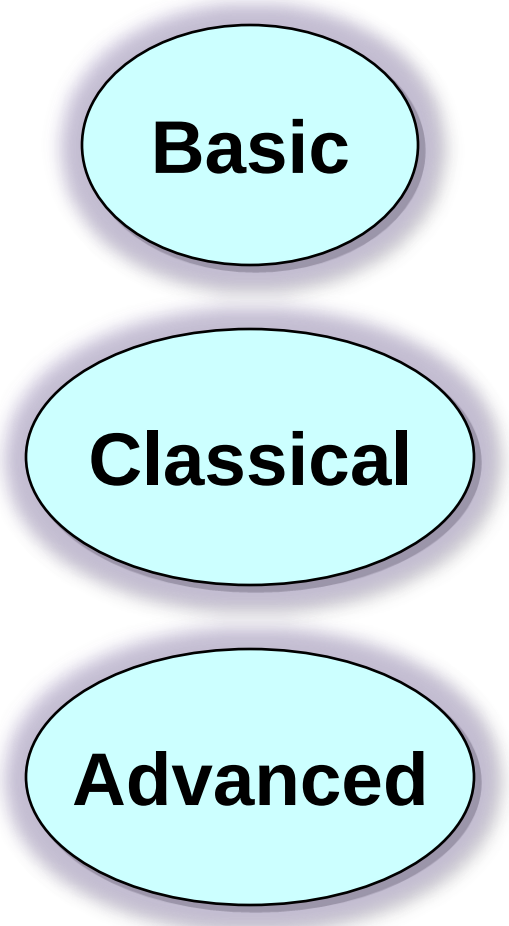
KPA
Insights through analytics

# Agenda

- **Visualizing Multivariate Data**
  - » Scatter plots
  - » Bubble plots
- **Multivariate Process Control**
  - » $T^2$ charts
  - » Two examples
- **Multivariate Data Analysis**
  - » Association rules
  - » The Italian case study



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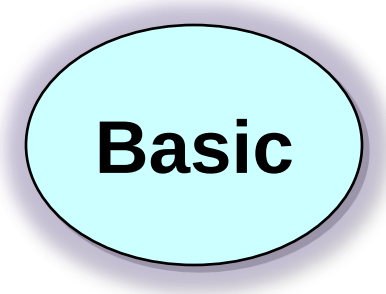


- **Multivariate Process Control**
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- **Multivariate Data Analysis**
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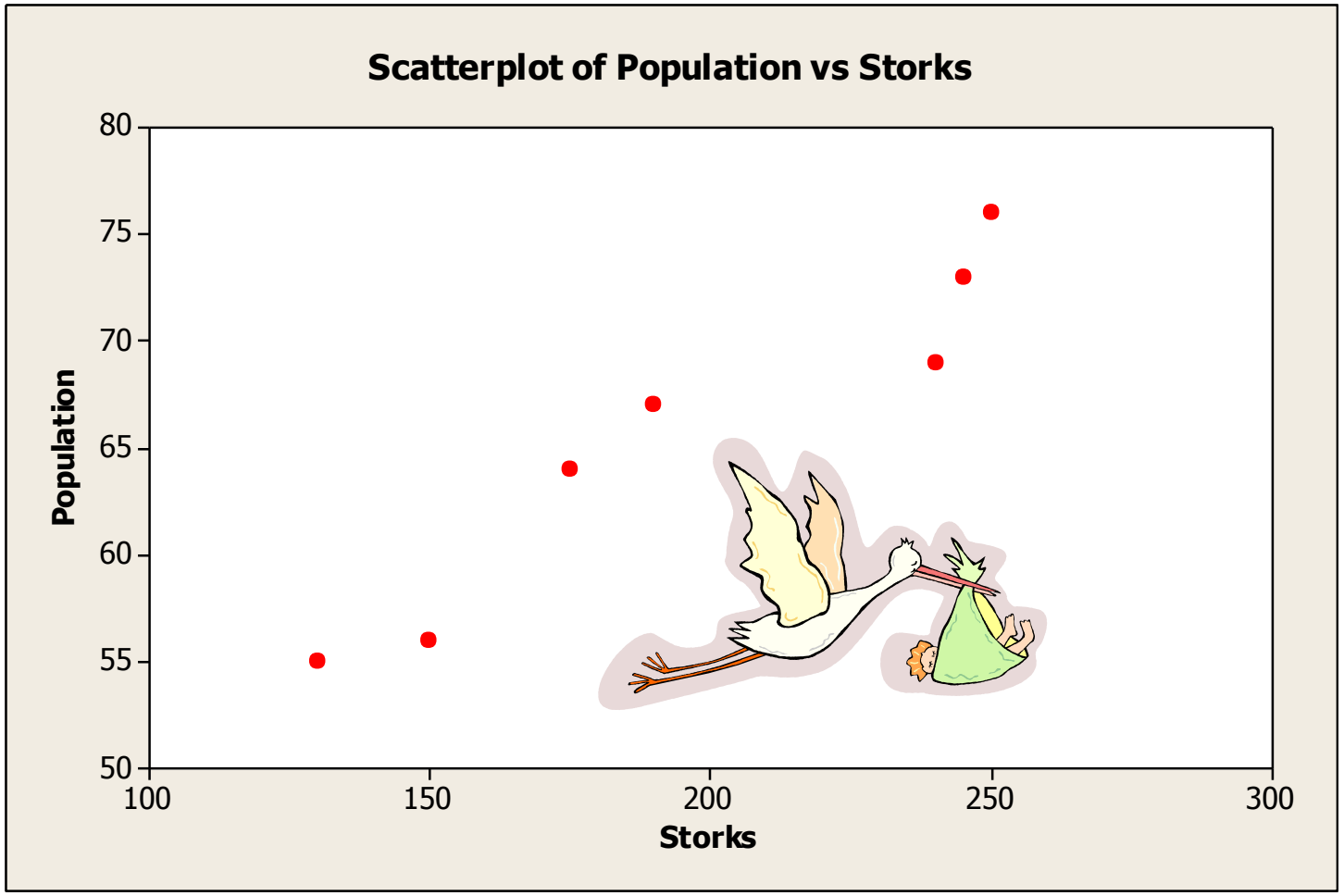
# Data in Two Dimensions

**The population of Oldenburg in Germany and the number of observed storks in 1930-1936***

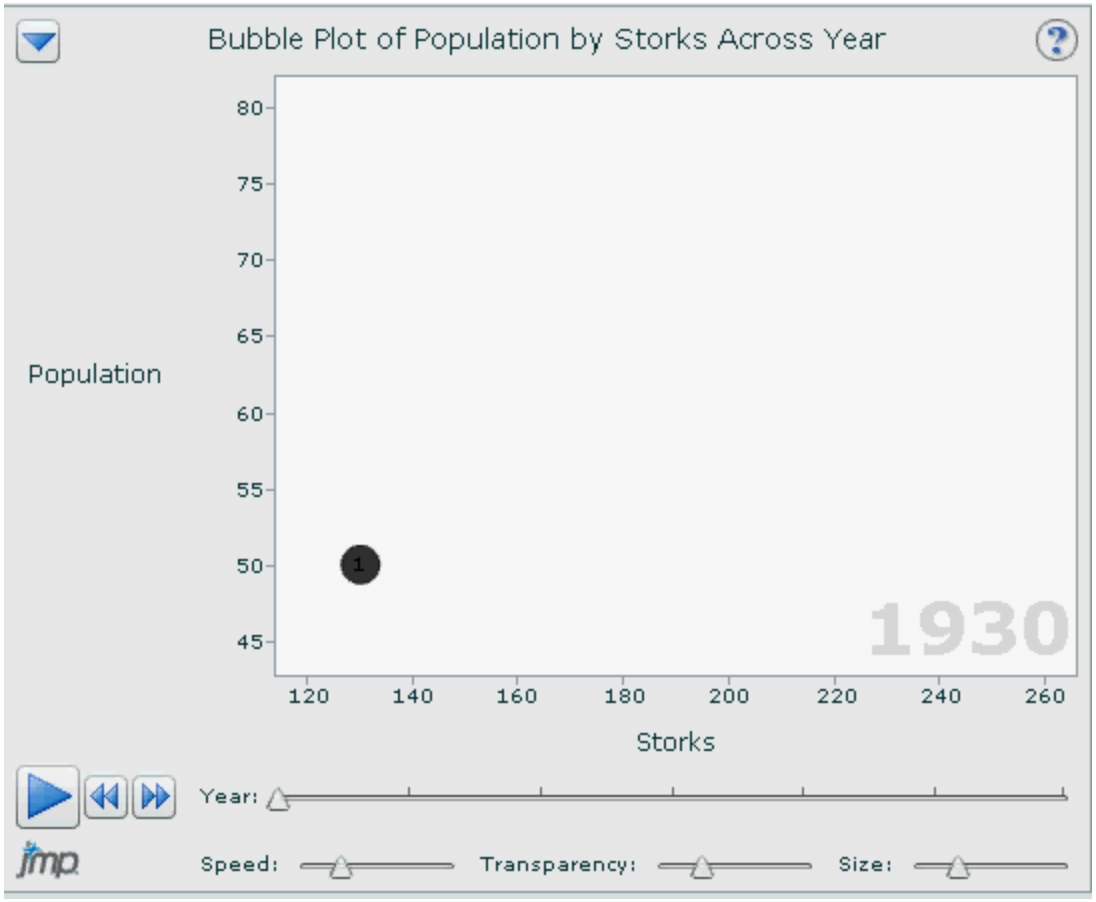
| Year | 1930 | 1931 | 1932 | 1933 | 1934 | 1935 | 1936 |
|---|---|---|---|---|---|---|---|
| Population in thousands | 50 | 52 | 64 | 67 | 69 | 73 | 76 |
| Number of storks | 130 | 150 | 175 | 190 | 240 | 245 | 250 |

* Box, Hunter and Hunter (1978) Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building, J. Wiley

# The Scatter Plot



# Bubble Plots



# Agenda

- Visualizing Multivariate Data
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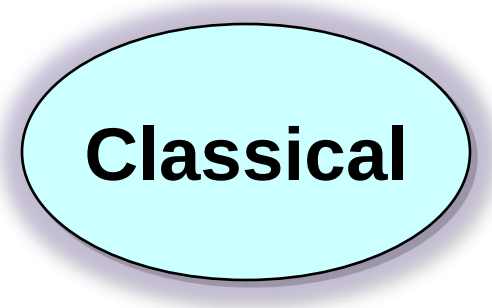
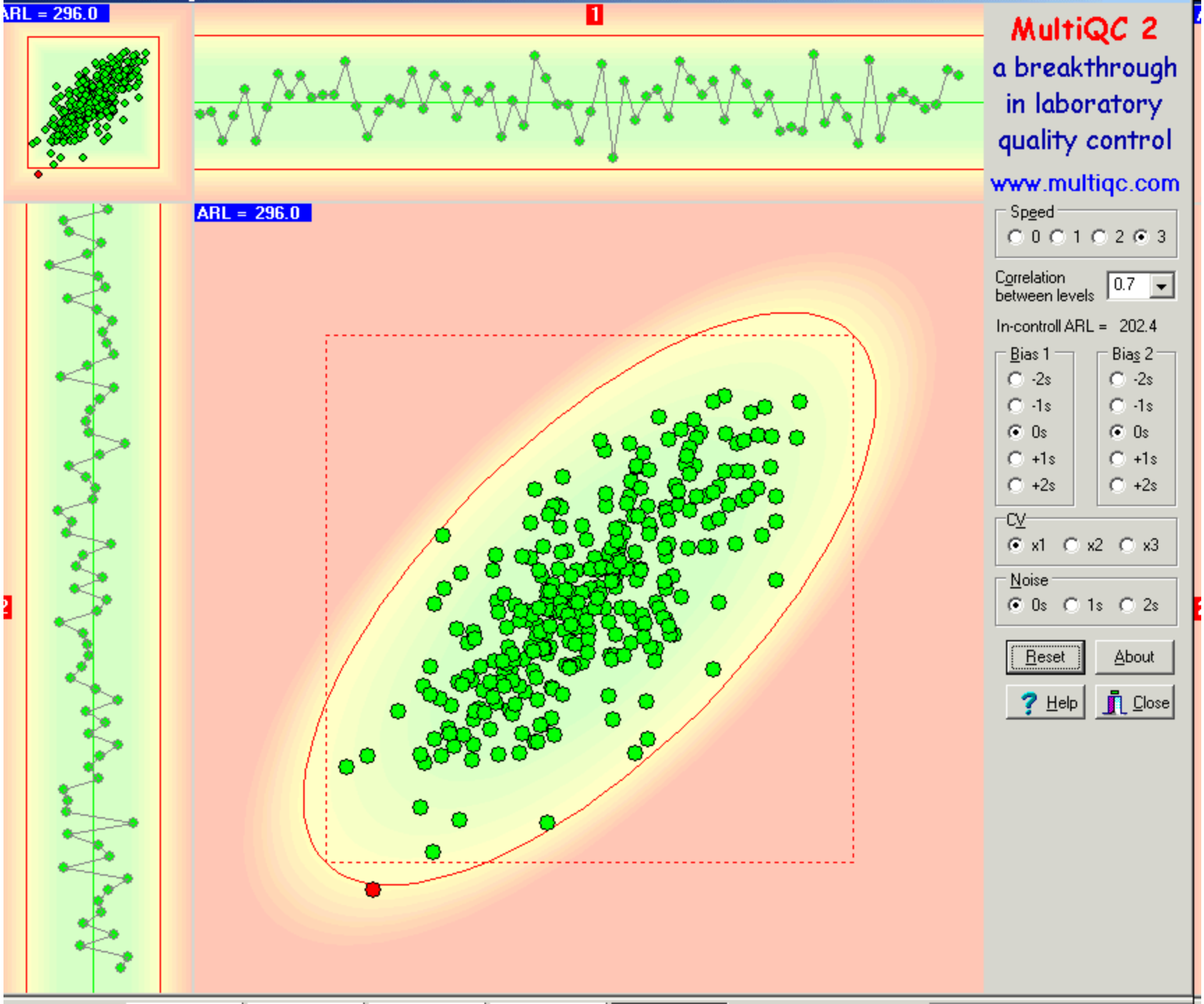
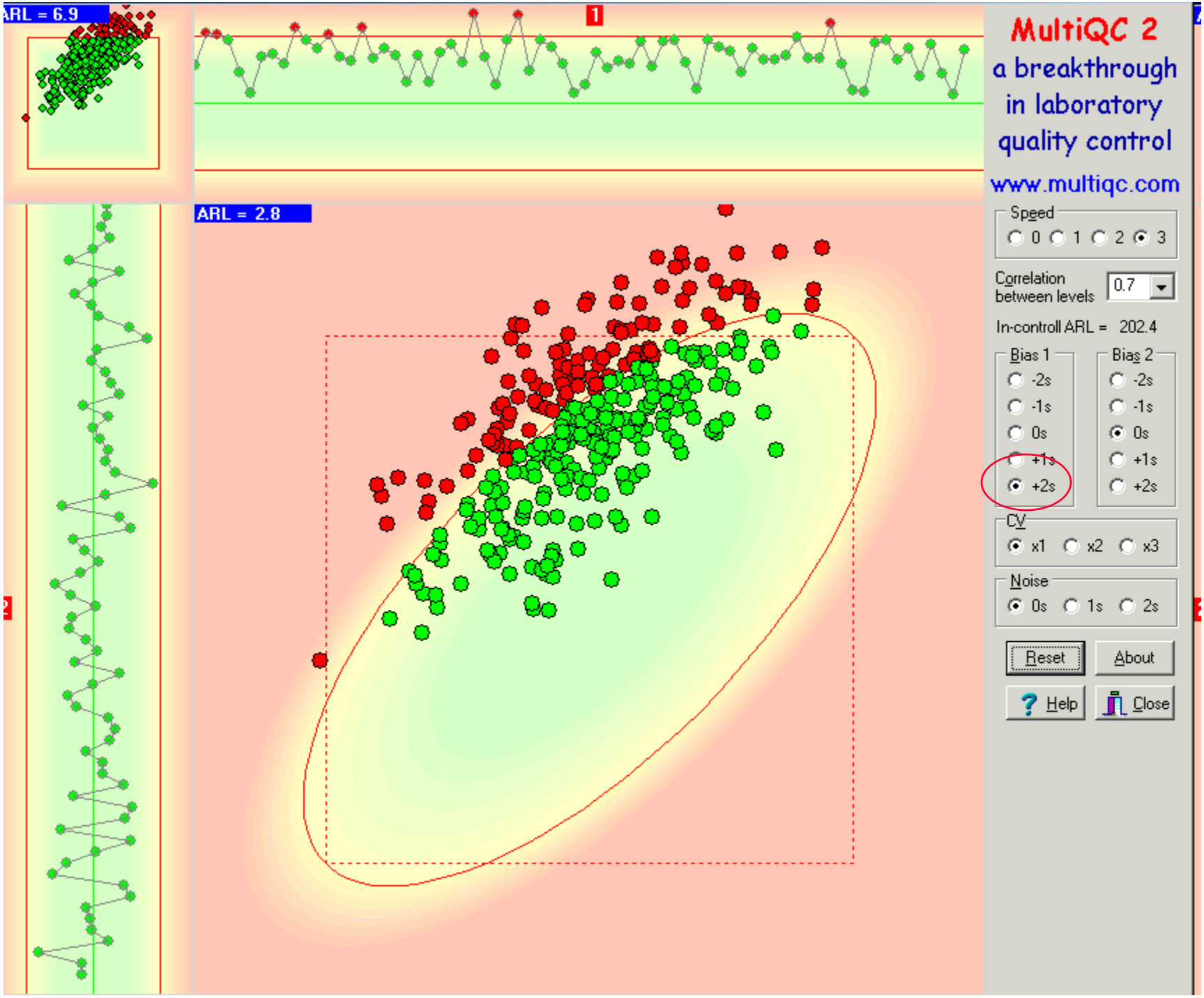



ARL = 296.0
1
MultiQC 2
a breakthrough
in laboratory
quality control
www.multiqc.com
ARL = 296.0
Speed
0 1 2 3
Correlation
between levels
0.7
In-controll ARL = 202.4
Bias 1
-2s
-1s
0s
+1s
+2s
Bias 2
-2s
-1s
0s
+1s
+2s
CV
x1 x2 x3
Noise
0s 1s 2s
Reset
About
Help
Close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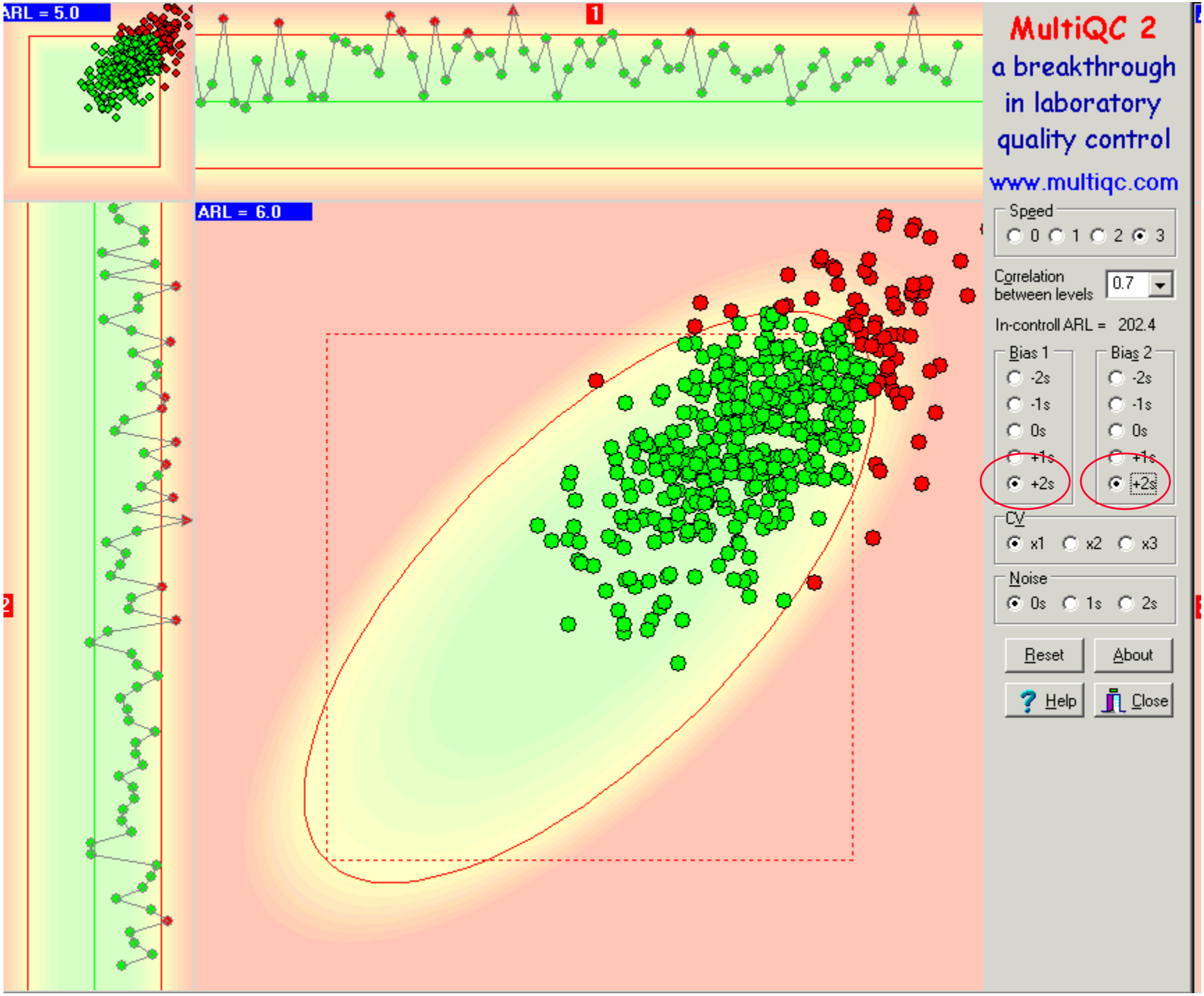
1
MultiQC 2
a breakthrough
in laboratory
quality control
www.multiqc.com
ARL = 2.8
Speed
0 1 2 3
Correlation
between levels
0.7
In-controll ARL = 202.4
Bias 1
-2s
-1s
0s
+1s
+2s
Bias 2
-2s
-1s
0s
+1s
+2s
CV
x1 x2 x3
Noise
0s 1s 2s
Reset
About
Help
Close


ARL = 5.0
1
ARL = 6.0
2
MultiQC 2
a breakthrough
in laboratory
quality control
www.multiqc.com
Speed
0 1 2 3
Correlation
between levels
0.7
In-controll ARL = 202.4
Bias 1
-2s
-1s
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+1s
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Bias 2
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0s
+1s
+2s
CV
x1 x2 x3
Noise
0s 1s 2s
Reset
About
Help
Close

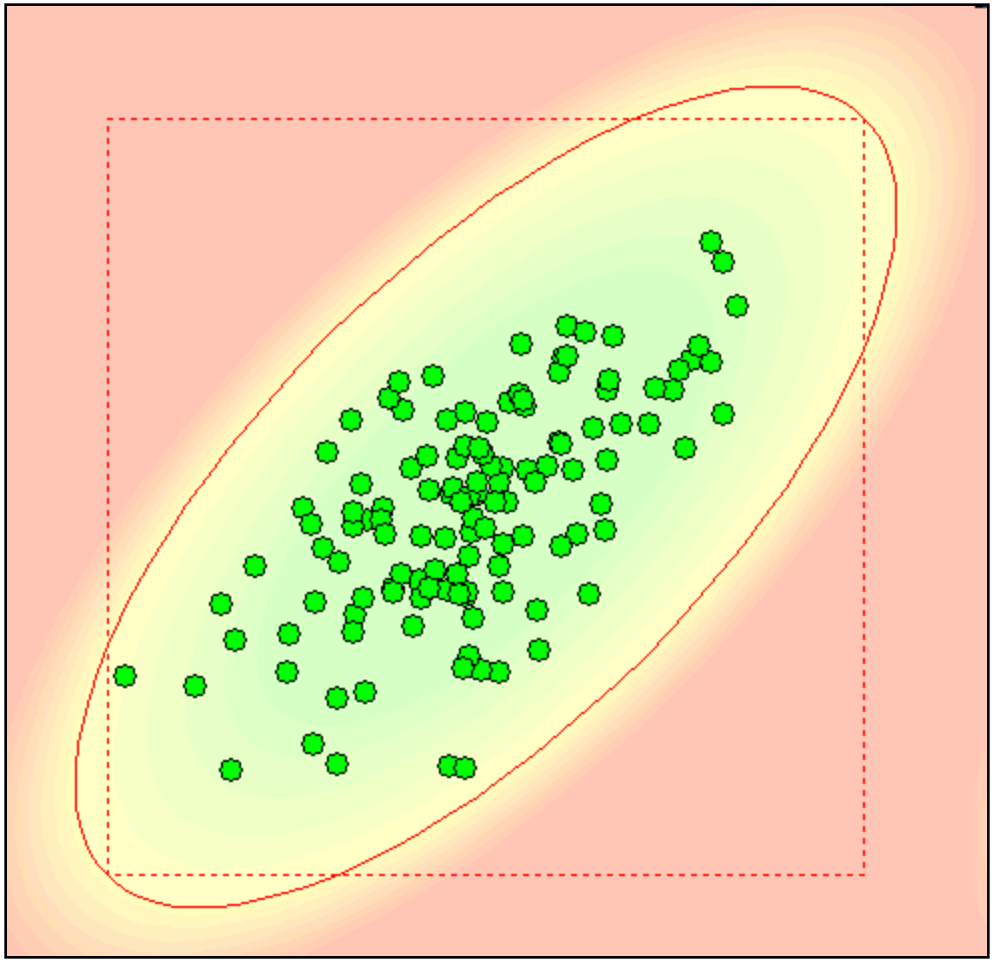
# Hotelling $T^2$ Control Charts

$$T^2 = n(\bar{\mathbf{x}} - \bar{\bar{\mathbf{x}}})'\mathbf{S}^{-1}(\bar{\mathbf{x}} - \bar{\bar{\mathbf{x}}})$$


Hotelling H. (1947). Multivariate quality control, illustrated by the air testing of sample bombsights in *Techniques of Statistical Analysis*, C. Eisenhart, M.W. Hastay and W.A. Wallis, McGraw-Hill: New York.

Jackson J.E.(1985), Multivariate quality control, Communications in Statistics: Theory and Methods, pp. 142657–2688.

Fuchs, C. and Kenett, R.S. (2007). "Multivariate Statistical Process Control", in *Encyclopedia of Statistics in Quality and Reliability*, Ruggeri, F., Kenett, R.S. and Faltin, F. (editors), Wiley.

# Hotelling $T^2$ Control Charts

Conditions affecting calculations and the degrees of freedom in the UCL*:

- **Internally derived reference sample**
- **Measurement units considered as individual data**
- Externally assigned targets
- Using an external reference sample
- Measurements units considered as batches

*Fuchs, C. and Kenett, R. (1998), *Multivariate Quality Control: Theory and application*, Quality and Reliability series volume 54, Marcel Dekker Inc.: New York.

# Hotelling $T^2$ Control Charts

## Internally derived reference sample

- **Phase I (process capability analysis):**

$$UCL = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{\alpha, p, mn-m-p+1}$$

$$LCL = 0$$

- **Phase II (on-going process control):**

$$UCL = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha, p, mn-m-p+1}$$

$$LCL = 0$$

# Hotelling $T^2$ Control Charts

## Measurement units considered as individual data

**If the sample taken is of size one (n=1) then**

$$T^2 = \left(\mathbf{x} - \bar{\mathbf{x}}\right)' \mathbf{S}^{-1} \left(\mathbf{x} - \bar{\mathbf{x}}\right)$$

**Phase II (on-going process control):**

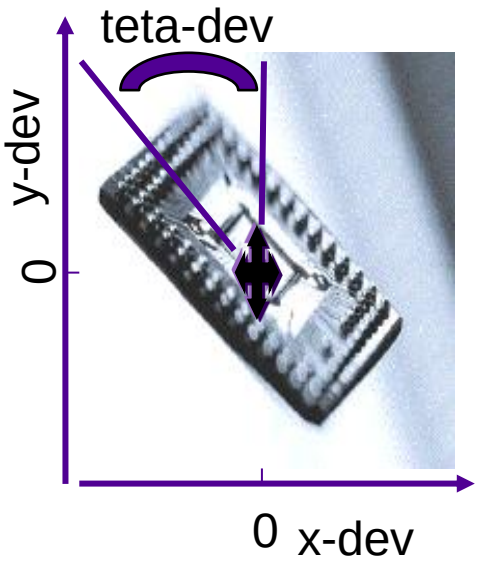
$$UCL = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha, p, m-p}$$

$$LCL = 0$$

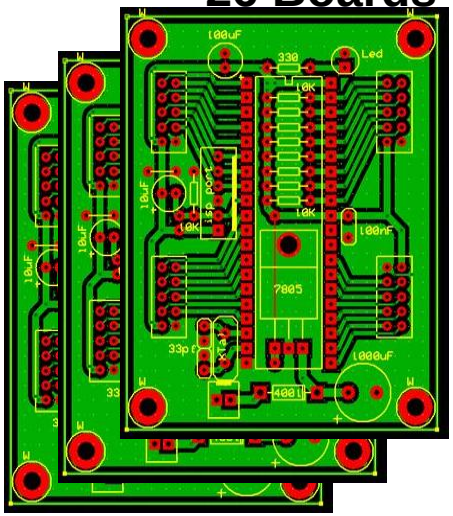
# The Component Placement Data

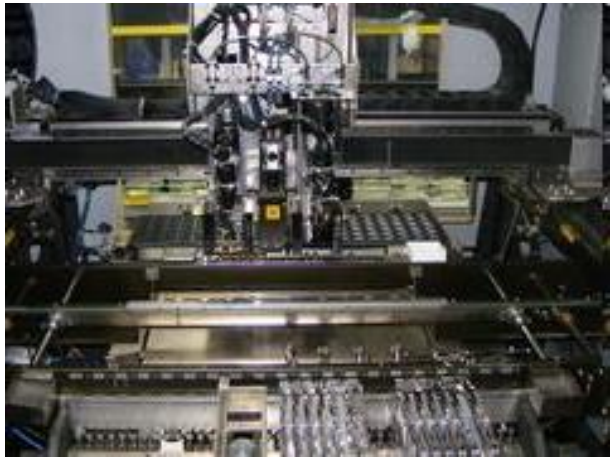
| Board_Nu | x-dev | y-dev | teta-dev |
|---|---|---|---|
| 1 | -0.00128 | -0.00183 | 0.00732 |
| 1 | -0.00092 | -0.00142 | 0.06317 |
| 1 | -0.00104 | -0.00174 | 0.04221 |
| 1 | -0.00271 | -0.00120 | 0.04010 |
| 1 | -0.00174 | -0.00230 | 0.03069 |
| 1 | -0.00050 | -0.00157 | 0.03451 |
| 1 | -0.00138 | -0.00235 | -0.00775 |
| 1 | -0.00242 | -0.00208 | 0.02614 |
| 1 | -0.00104 | -0.00140 | -0.00245 |
| 1 | -0.00105 | -0.00115 | -0.00158 |
| 1 | -0.00081 | -0.00226 | -0.00002 |
| 1 | -0.00283 | -0.00145 | -0.00156 |
| 1 | -0.00101 | -0.00248 | 0.00057 |
| 1 | -0.00040 | -0.00132 | 0.00031 |
| 1 | -0.00091 | -0.00235 | 0.00148 |
| 1 | -0.00237 | -0.00130 | 0.00023 |
| 2 | -0.00116 | -0.00121 | 0.00742 |
| 2 | -0.00028 | -0.00082 | 0.03026 |
| 2 | -0.00072 | -0.00171 | 0.03988 |
| 2 | -0.00218 | -0.00071 | 0.02159 |

**16 Components**




**26 Boards**





* Kenett, R.S. and Zacks (1998), Modern Industrial Statistics: Design and control of quality and reliability, Duxbury Press

# Descriptive Statistics

**Descriptive Statistics: x-dev, y-dev, teta-dev**

| Variable | N | N* | Mean | SE Mean | TrMean | StDev | Variance |
|---|---|---|---|---|---|---|---|
| x-dev | 416 | 0 | 0.000912 | 0.0000839 | 0.000929 | 0.001711 | 0.00000293 |
| y-dev | 416 | 0 | -0.000588 | 0.0000558 | -0.000592 | 0.001138 | 0.00000130 |
| teta-dev | 416 | 0 | 0.01627 | 0.00132 | 0.01458 | 0.02699 | 0.000728 |

| Variable | CoefVar | Minimum | Q1 | Median | Q3 | Maximum |
|---|---|---|---|---|---|---|
| x-dev | 187.50 | -0.002860 | -0.000753 | 0.001655 | 0.002365 | 0.004560 |
| y-dev | -193.63 | -0.003160 | -0.001510 | -0.000535 | 0.000237 | 0.002920 |
| teta-dev | 165.85 | -0.07792 | -0.000765 | 0.00110 | 0.03339 | 0.12981 |

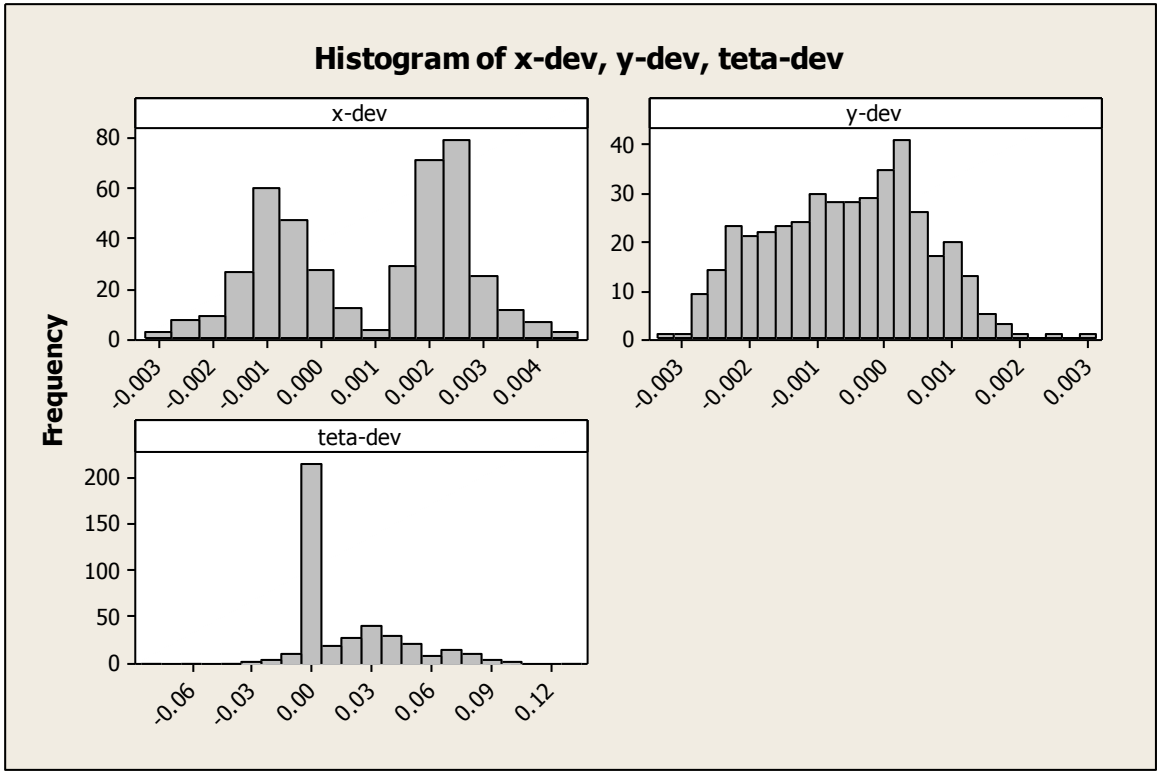
# Descriptive Statistics

**Correlations: x-dev, y-dev, teta-dev**

| | x-dev | y-dev |
|---|---|---|
| y-dev | **0.558** | |
| | 0.000 | |
| teta-dev | 0.079 | 0.109 |
| | 0.110 | 0.026 |

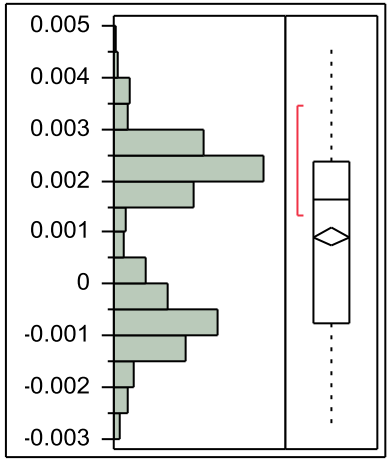
Cell Contents: Pearson correlation
P-Value

# Histograms


Histogram of x-dev, y-dev, teta-dev
x-dev
y-dev
teta-dev
Frequency
80
60
40
20
0
-0.003
-0.002
-0.001
0.000
0.001
0.002
0.003
0.004
40
30
20
10
0
200
150
100
50
-0.06
-0.03
0.00
0.03
0.06
0.09
0.12

## Distributions

### x-dev



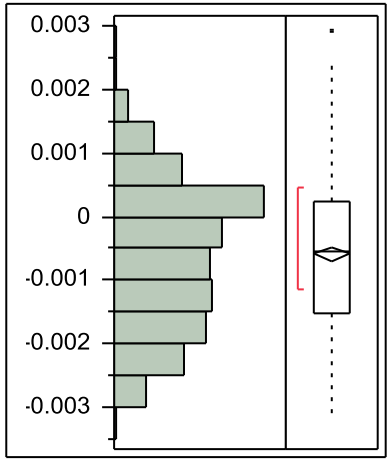

#### Quantiles

| | | |
|---|---|---|
| 100.0% | maximum | 0.0046 |
| 99.5% | | 0.0043 |
| 97.5% | | 0.0036 |
| 90.0% | | 0.0028 |
| 75.0% | quartile | 0.0024 |
| 50.0% | median | 0.0017 |
| 25.0% | quartile | -0.0008 |
| 10.0% | | -0.0013 |
| 2.5% | | -0.0022 |
| 0.5% | | -0.0028 |
| 0.0% | minimum | -0.0029 |

#### Moments

| | |
|---|---|
| Mean | 0.0009124 |
| Std Dev | 0.0017108 |
| Std Err Mean | 8.3881e-5 |
| Upper 95% Mean | 0.0010773 |
| Lower 95% Mean | 0.0007475 |
| N | 416 |

### y-dev



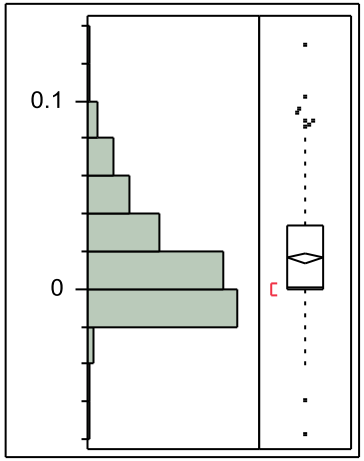

#### Quantiles

| | | |
|---|---|---|
| 100.0% | maximum | 0.0029 |
| 99.5% | | 0.0024 |
| 97.5% | | 0.0015 |
| 90.0% | | 0.00089 |
| 75.0% | quartile | 0.00024 |
| 50.0% | median | -0.0005 |
| 25.0% | quartile | -0.0015 |
| 10.0% | | -0.0022 |
| 2.5% | | -0.0027 |
| 0.5% | | -0.0029 |
| 0.0% | minimum | -0.0032 |

#### Moments

| | |
|---|---|
| Mean | -0.000588 |
| Std Dev | 0.0011382 |
| Std Err Mean | 0.0000558 |
| Upper 95% Mean | -0.000478 |
| Lower 95% Mean | -0.000698 |
| N | 416 |

### teta-dev




#### Quantiles

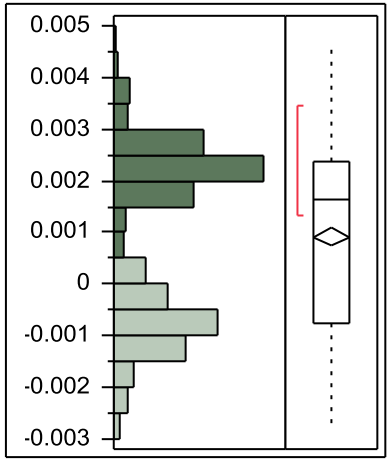
| | | |
|---|---|---|
| 100.0% | maximum | 0.1298 |
| 99.5% | | 0.1009 |
| 97.5% | | 0.0818 |
| 90.0% | | 0.0549 |
| 75.0% | quartile | 0.0334 |
| 50.0% | median | 0.0011 |
| 25.0% | quartile | -0.0008 |
| 10.0% | | -0.0024 |
| 2.5% | | -0.0151 |
| 0.5% | | -0.0579 |
| 0.0% | minimum | -0.0779 |

#### Moments

| | |
|---|---|
| Mean | 0.0162715 |
| Std Dev | 0.0269863 |
| Std Err Mean | 0.0013231 |
| Upper 95% Mean | 0.0188723 |
| Lower 95% Mean | 0.0136706 |
| N | 416 |

## Distributions

### x-dev



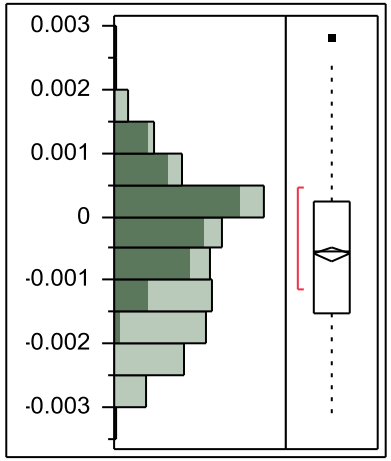

#### Quantiles

| | | |
|---|---|---|
| 100.0% | maximum | 0.0046 |
| 99.5% | | 0.0043 |
| 97.5% | | 0.0036 |
| 90.0% | | 0.0028 |
| 75.0% | quartile | 0.0024 |
| 50.0% | median | 0.0017 |
| 25.0% | quartile | -0.0008 |
| 10.0% | | -0.0013 |
| 2.5% | | -0.0022 |
| 0.5% | | -0.0028 |
| 0.0% | minimum | -0.0029 |

#### Moments

| | |
|---|---|
| Mean | 0.0009124 |
| Std Dev | 0.0017108 |
| Std Err Mean | 8.3881e-5 |
| Upper 95% Mean | 0.0010773 |
| Lower 95% Mean | 0.0007475 |
| N | 416 |

### y-dev



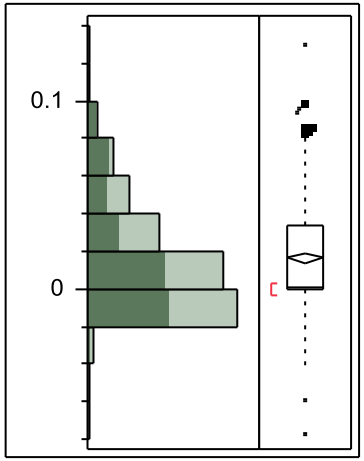

#### Quantiles

| | | |
|---|---|---|
| 100.0% | maximum | 0.0029 |
| 99.5% | | 0.0024 |
| 97.5% | | 0.0015 |
| 90.0% | | 0.00089 |
| 75.0% | quartile | 0.00024 |
| 50.0% | median | -0.0005 |
| 25.0% | quartile | -0.0015 |
| 10.0% | | -0.0022 |
| 2.5% | | -0.0027 |
| 0.5% | | -0.0029 |
| 0.0% | minimum | -0.0032 |

#### Moments

| | |
|---|---|
| Mean | -0.000588 |
| Std Dev | 0.0011382 |
| Std Err Mean | 0.0000558 |
| Upper 95% Mean | -0.000478 |
| Lower 95% Mean | -0.000698 |
| N | 416 |

### teta-dev



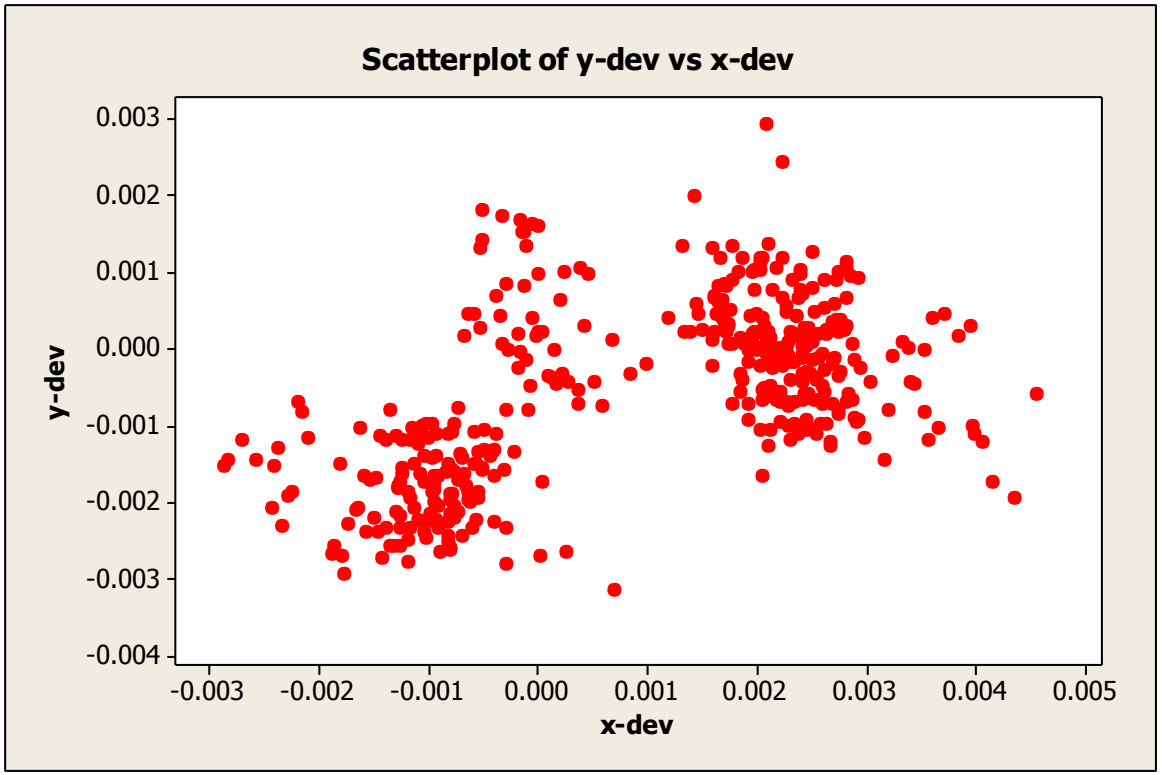

#### Quantiles

| | | |
|---|---|---|
| 100.0% | maximum | 0.1298 |
| 99.5% | | 0.1009 |
| 97.5% | | 0.0818 |
| 90.0% | | 0.0549 |
| 75.0% | quartile | 0.0334 |
| 50.0% | median | 0.0011 |
| 25.0% | quartile | -0.0008 |
| 10.0% | | -0.0024 |
| 2.5% | | -0.0151 |
| 0.5% | | -0.0579 |
| 0.0% | minimum | -0.0779 |

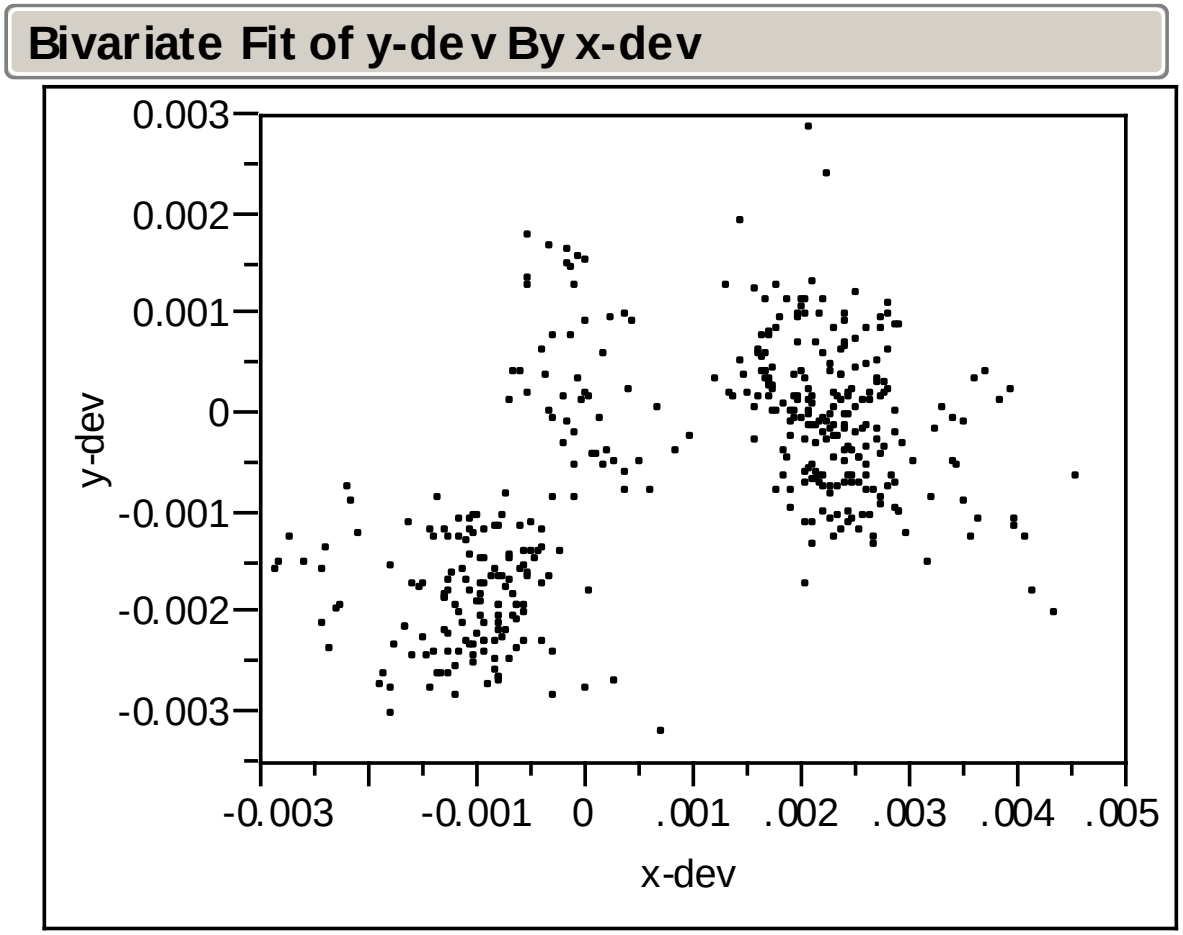
#### Moments

| | |
|---|---|
| Mean | 0.0162715 |
| Std Dev | 0.0269863 |
| Std Err Mean | 0.0013231 |
| Upper 95% Mean | 0.0188723 |
| Lower 95% Mean | 0.0136706 |
| N | 416 |

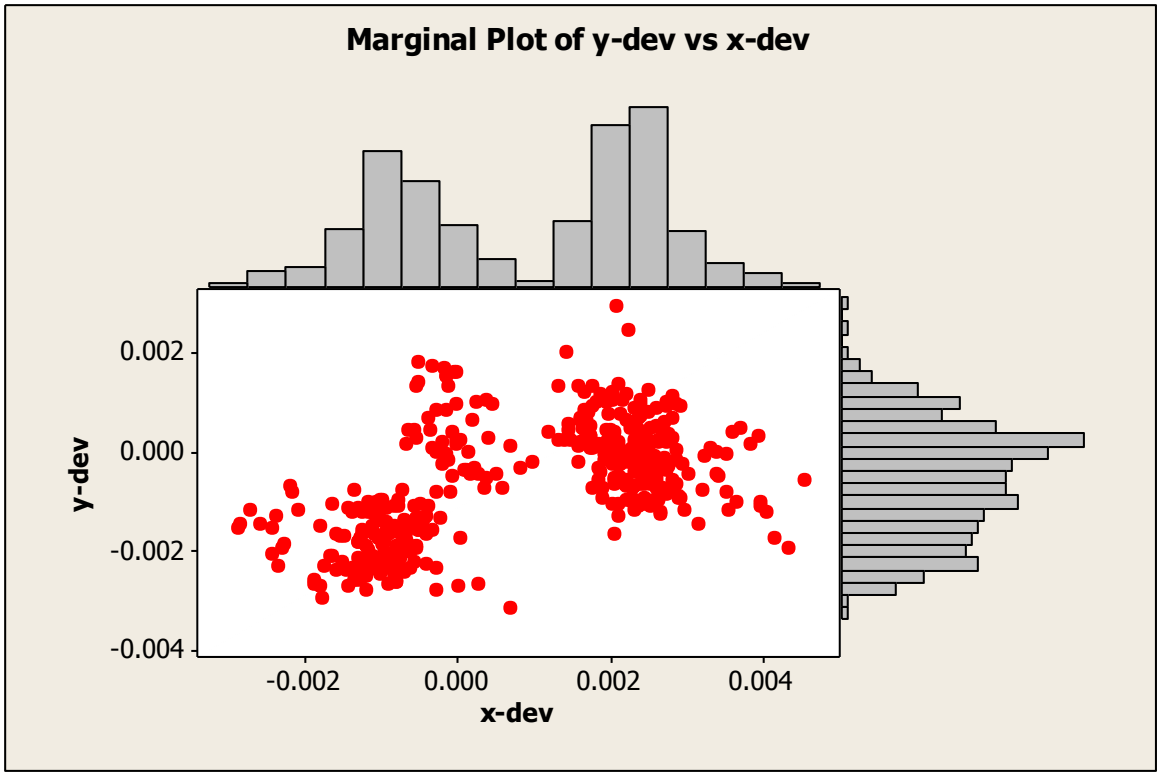
# Scatter Plot of y-dev vs. x-dev



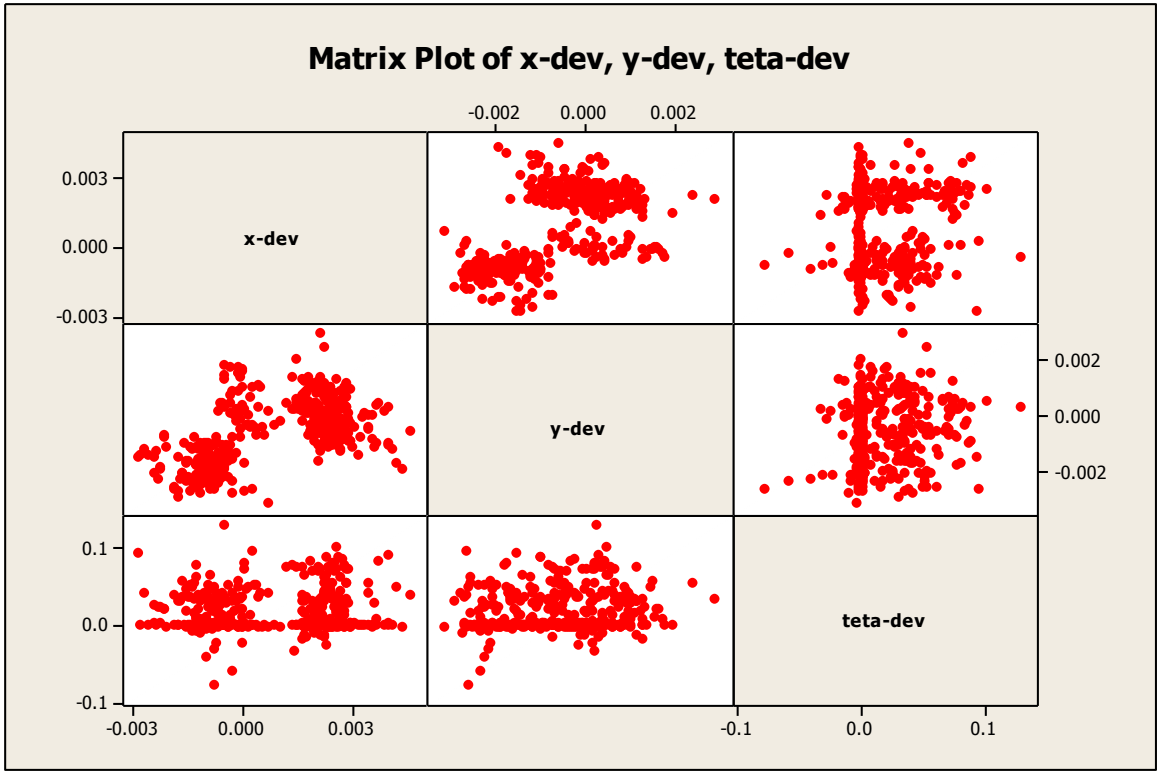
# Scatter Plot of y-dev vs. x-dev



# Scatter Plot of y-dev vs. x-dev



# Matrix Plot


Matrix Plot of x-dev, y-dev, teta-dev
x-dev
y-dev
teta-dev
-0.002
0.000
0.002
0.003
0.000
-0.003
0.002
0.000
-0.002
0.1
0.0
-0.1
-0.003
0.000
0.003
-0.1
0.0
0.1

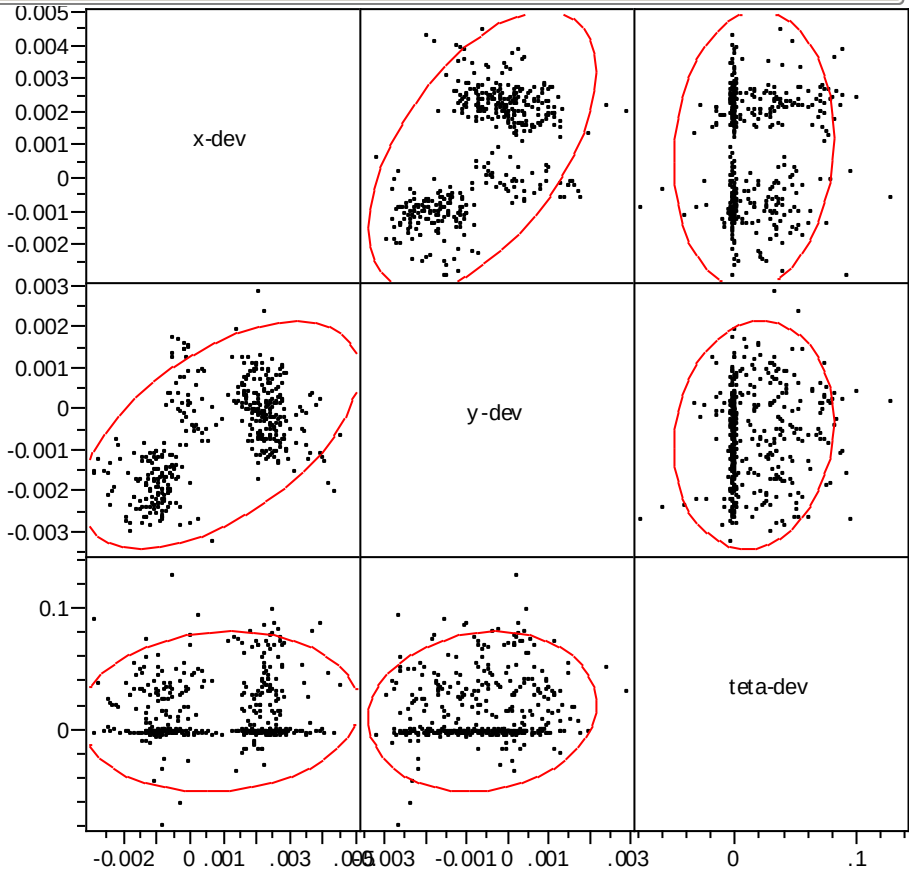
# Matrix Plot

**Multivariate**

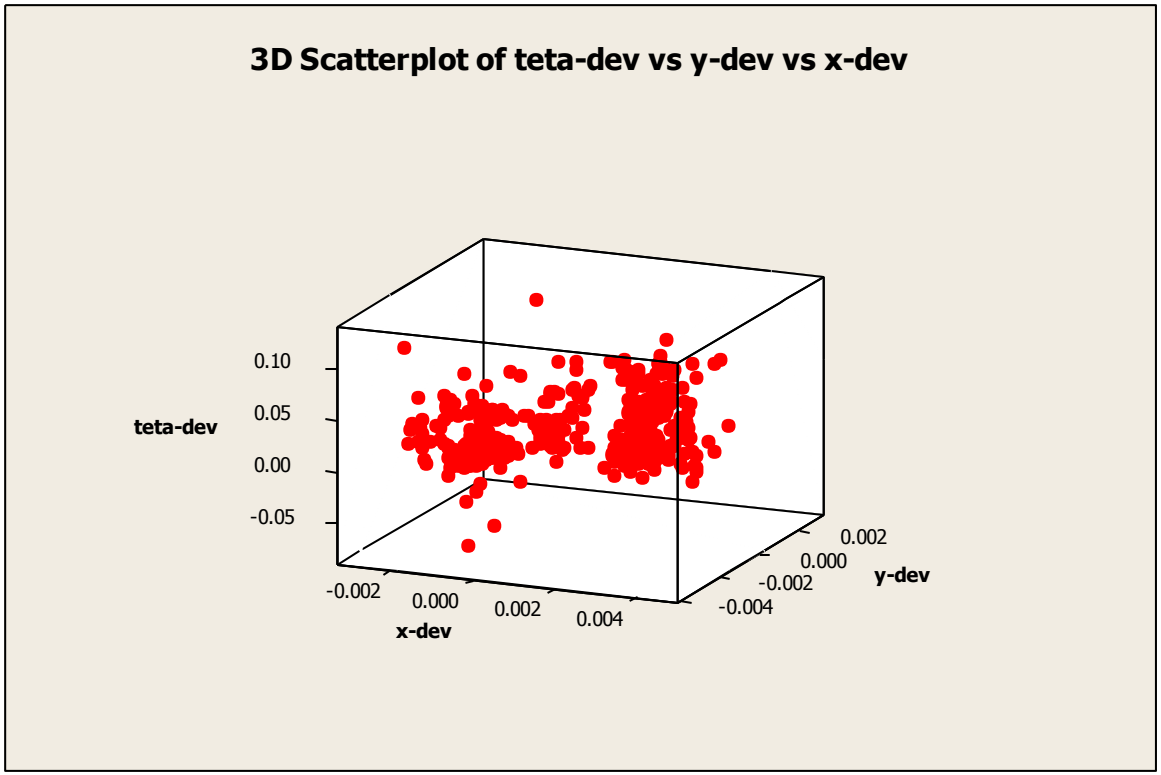
**Correlations**

| | x-dev | y-dev | teta-dev |
|---|---|---|---|
| x-dev | 1.0000 | 0.5583 | 0.0785 |
| y-dev | 0.5583 | 1.0000 | 0.1088 |
| teta-dev | 0.0785 | 0.1088 | 1.0000 |

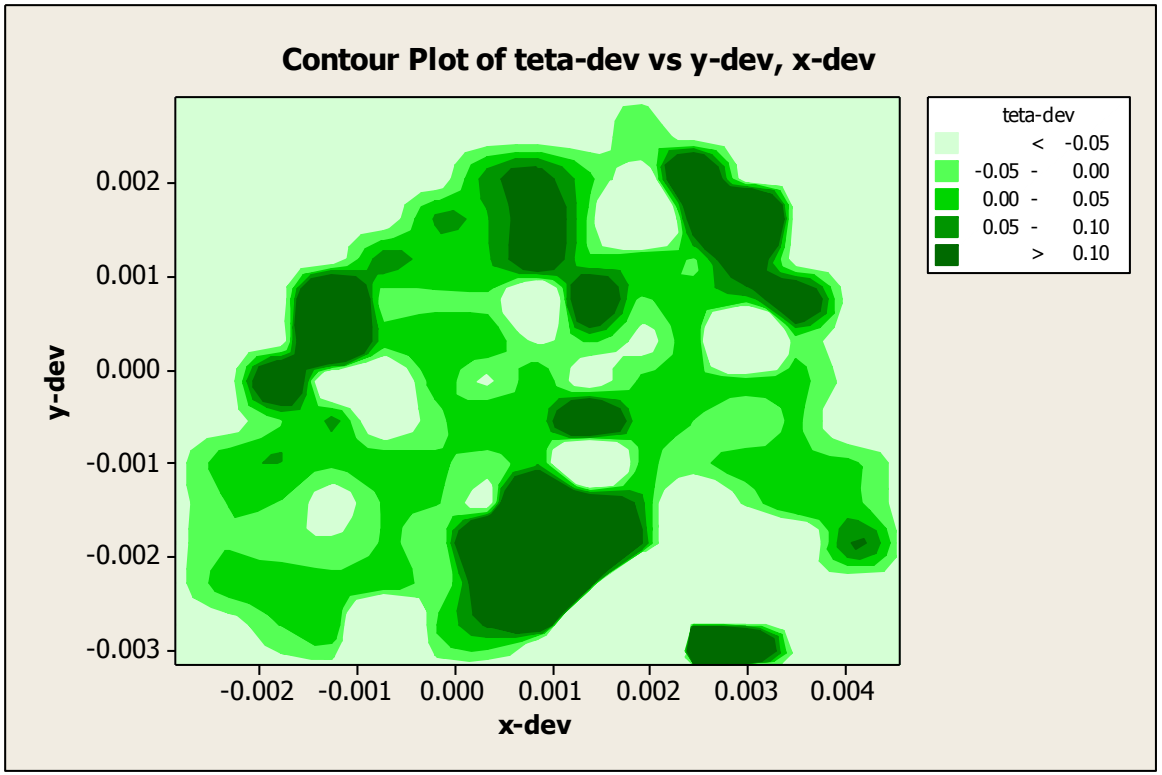
**Scatterplot Matrix**



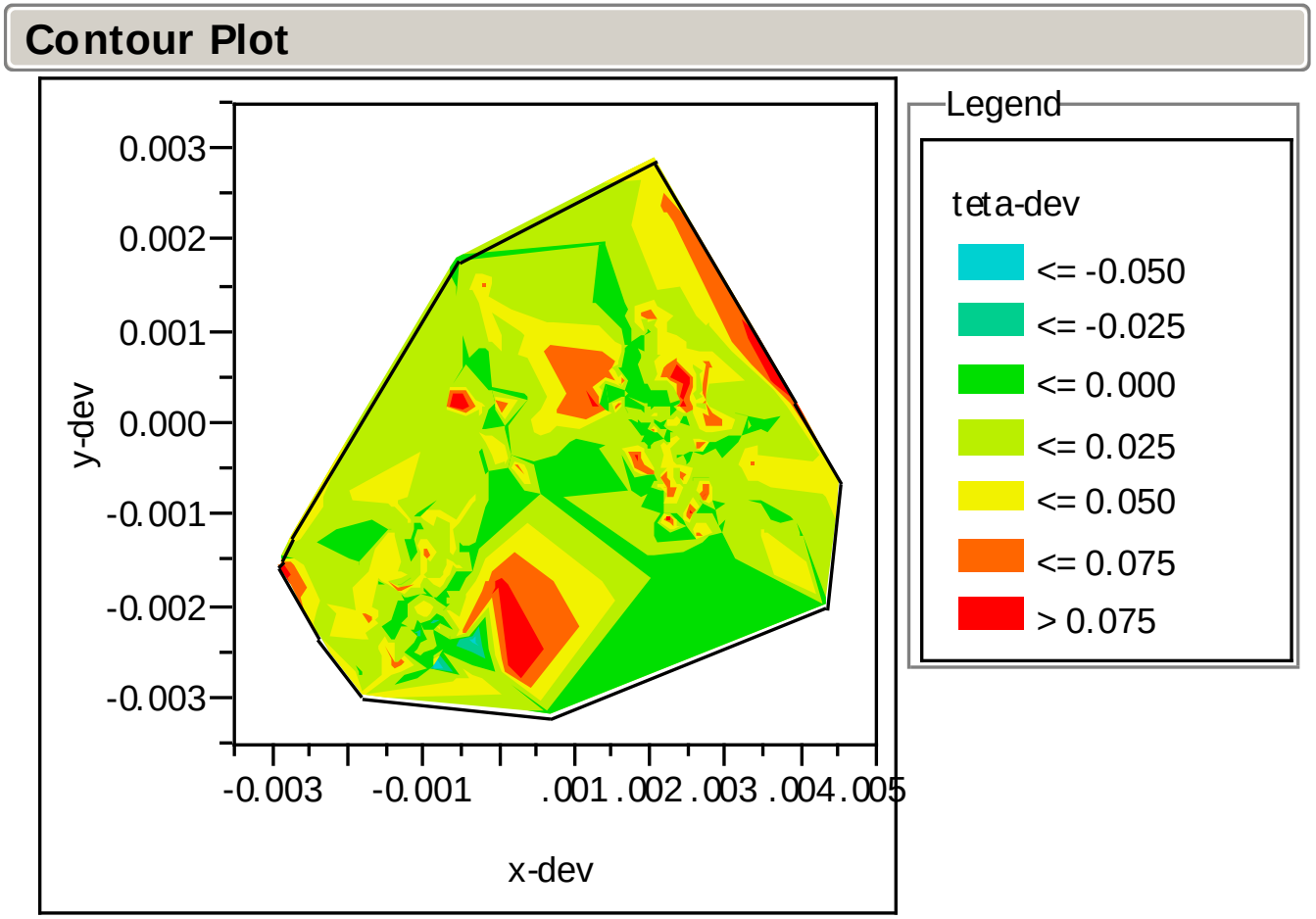
# 3D Scatter Plot



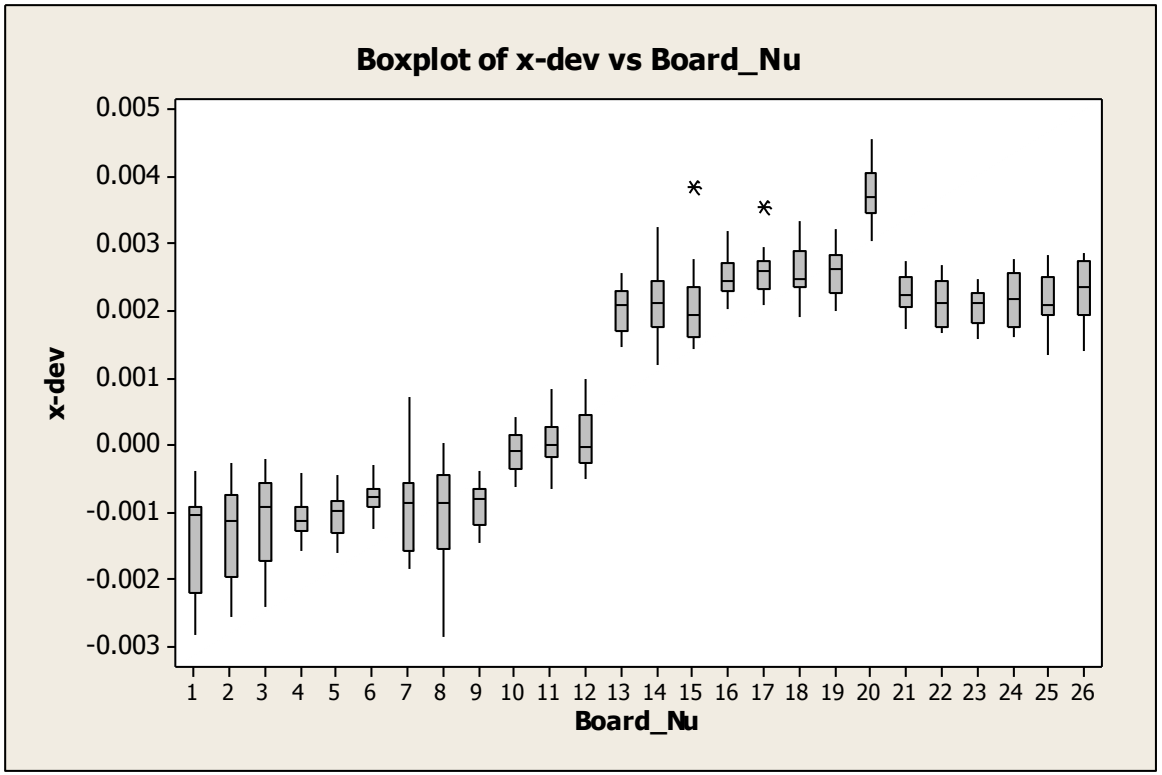
# Contour Plots



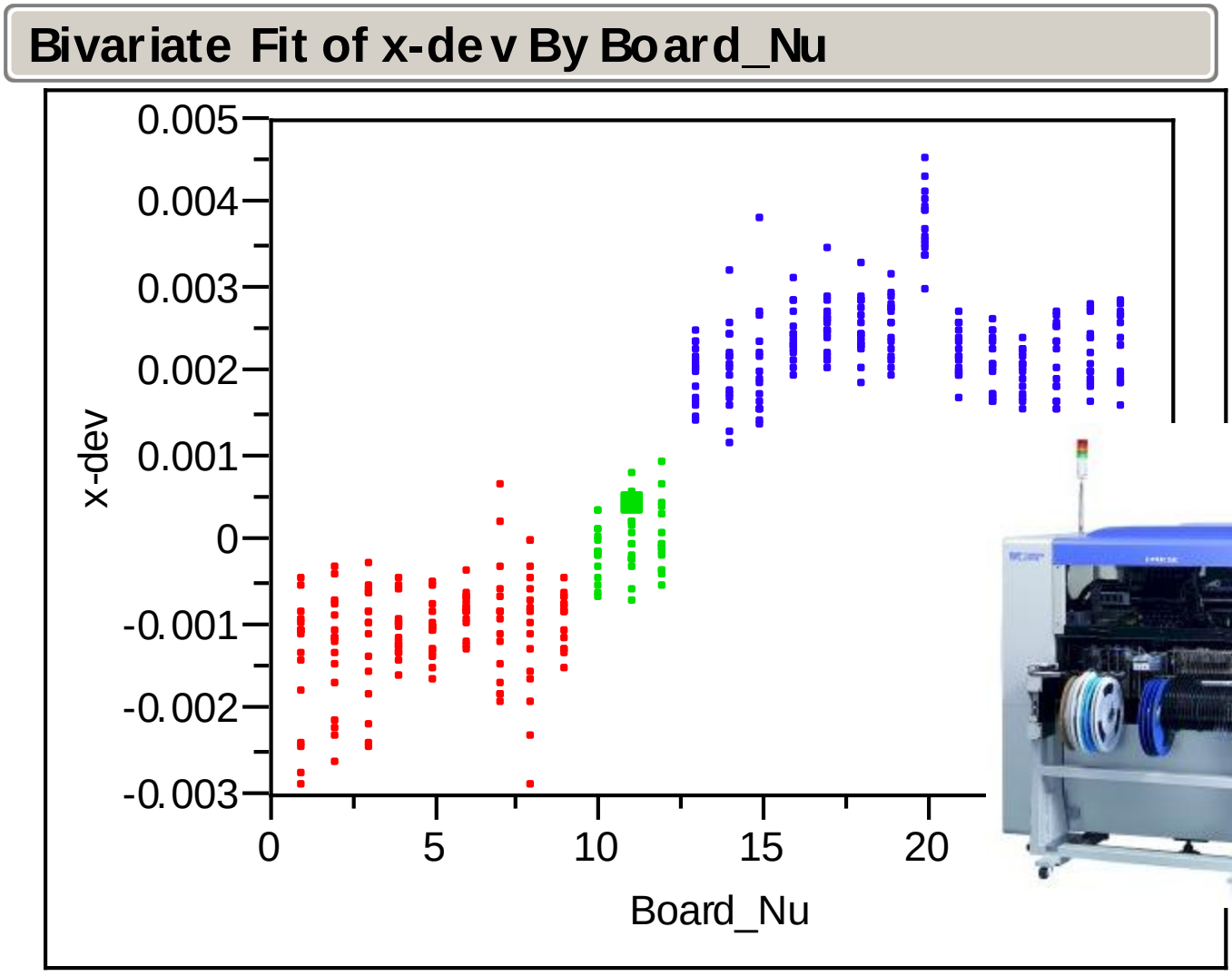
# Contour Plots



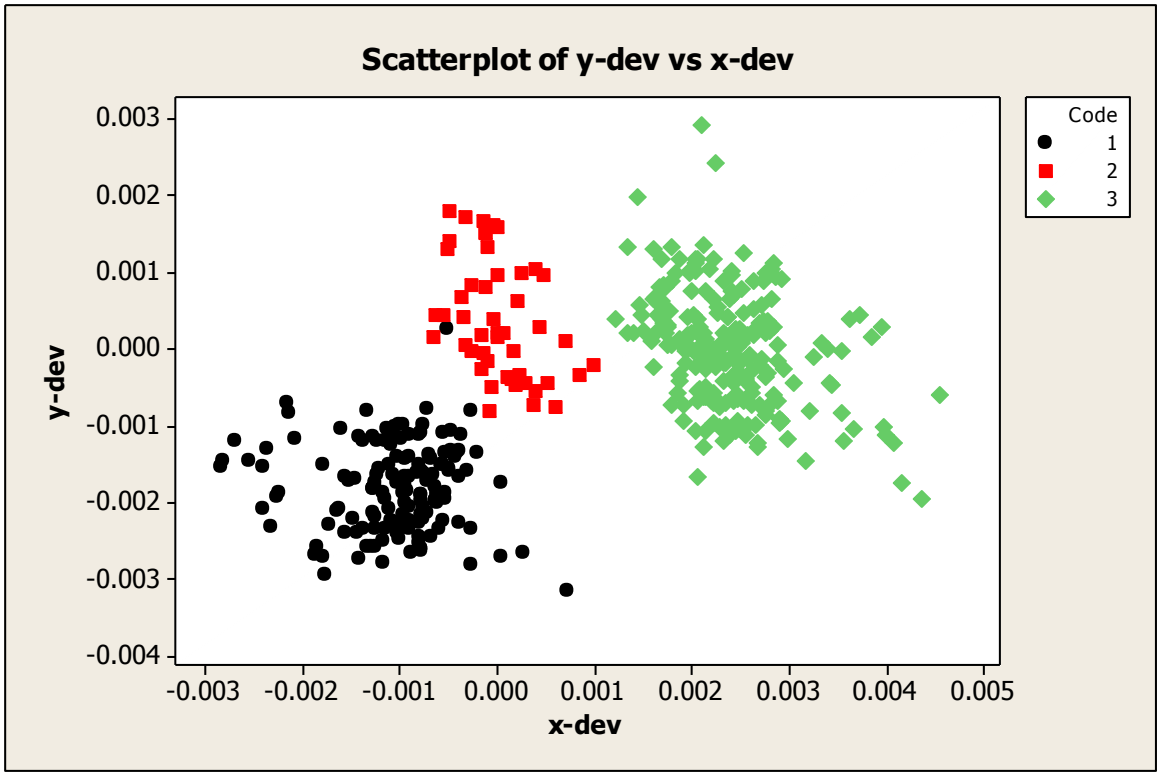
# Boxplots of x-dev vs. board #



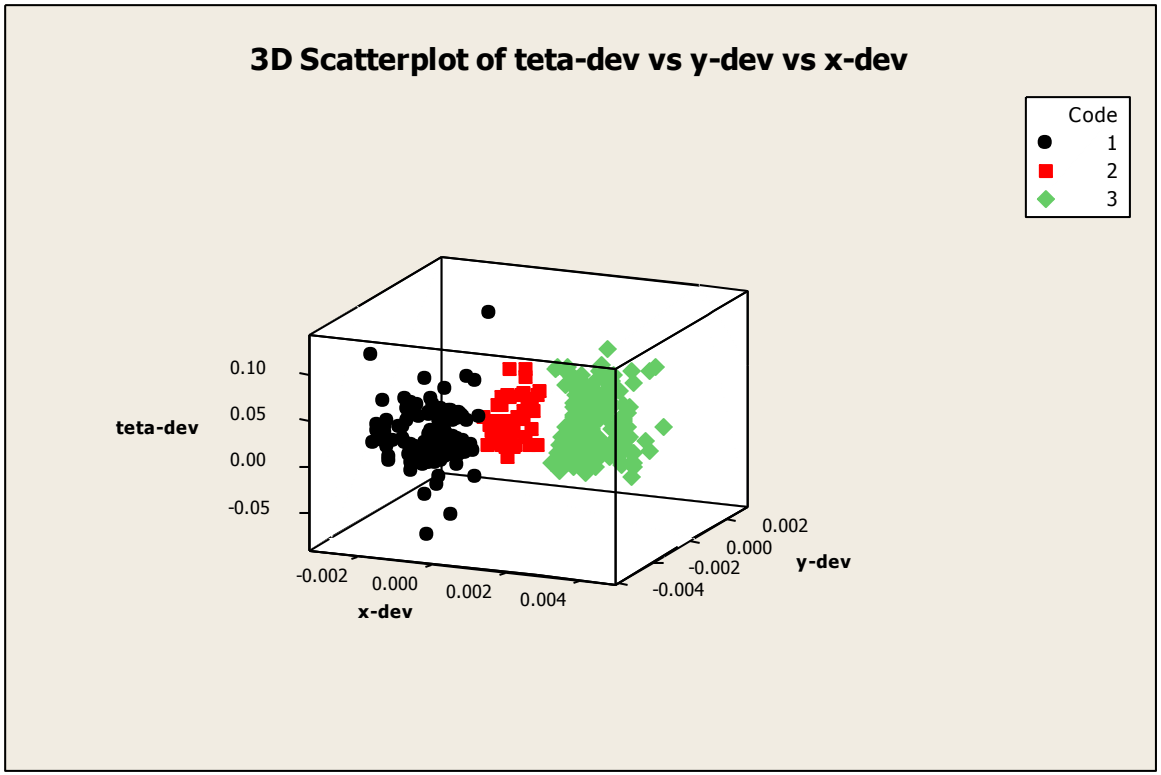
# Boxplots of x-dev vs. board #



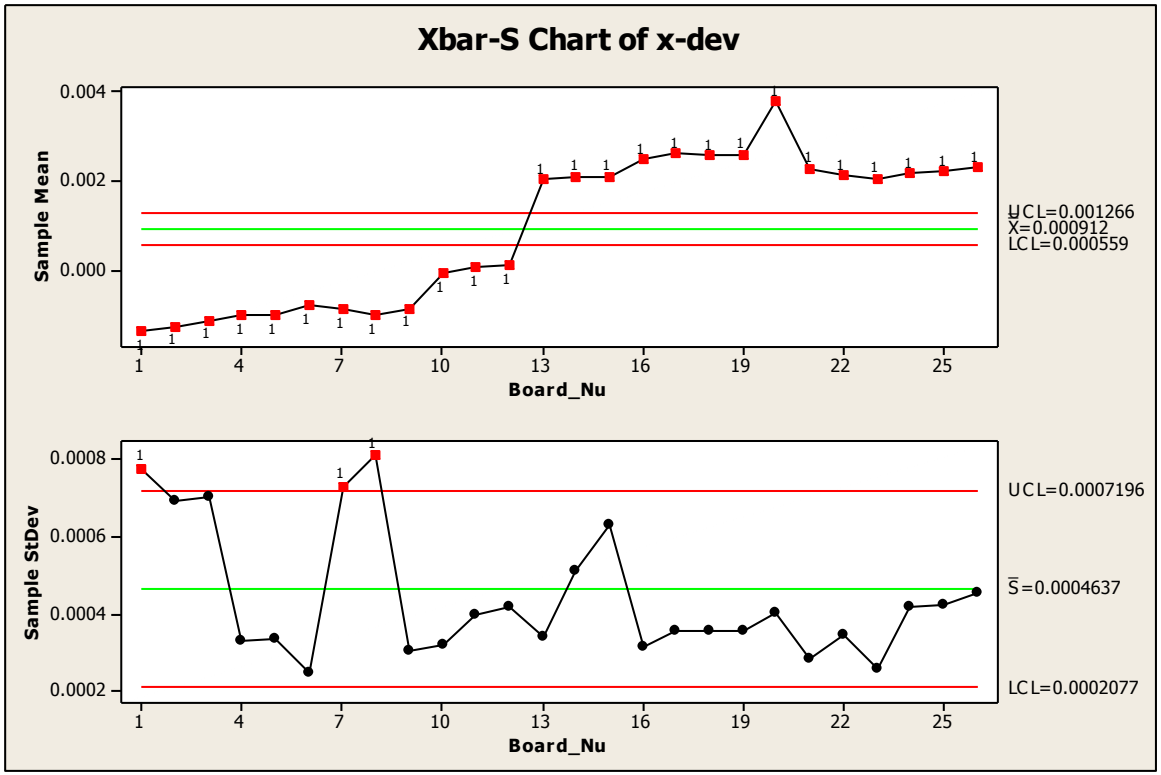
# Coded Scatter Plot



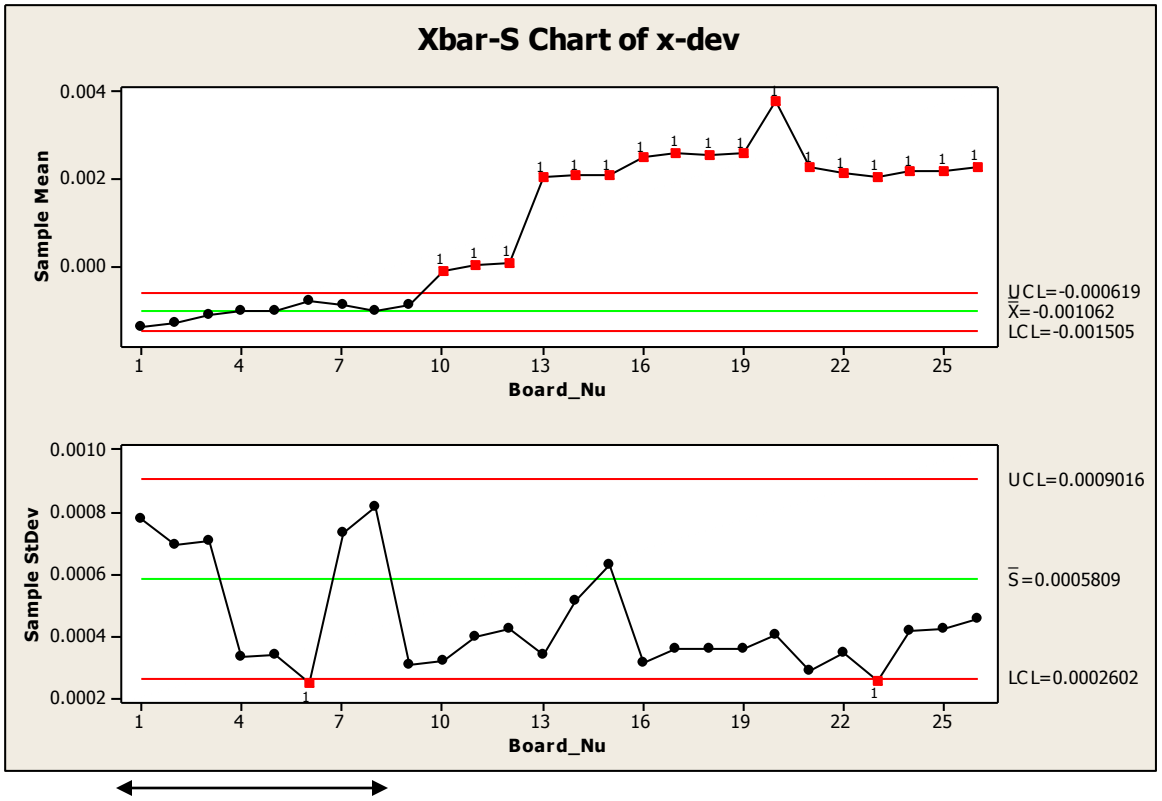
# Coded 3D Plot



# Control Chart for x-dev

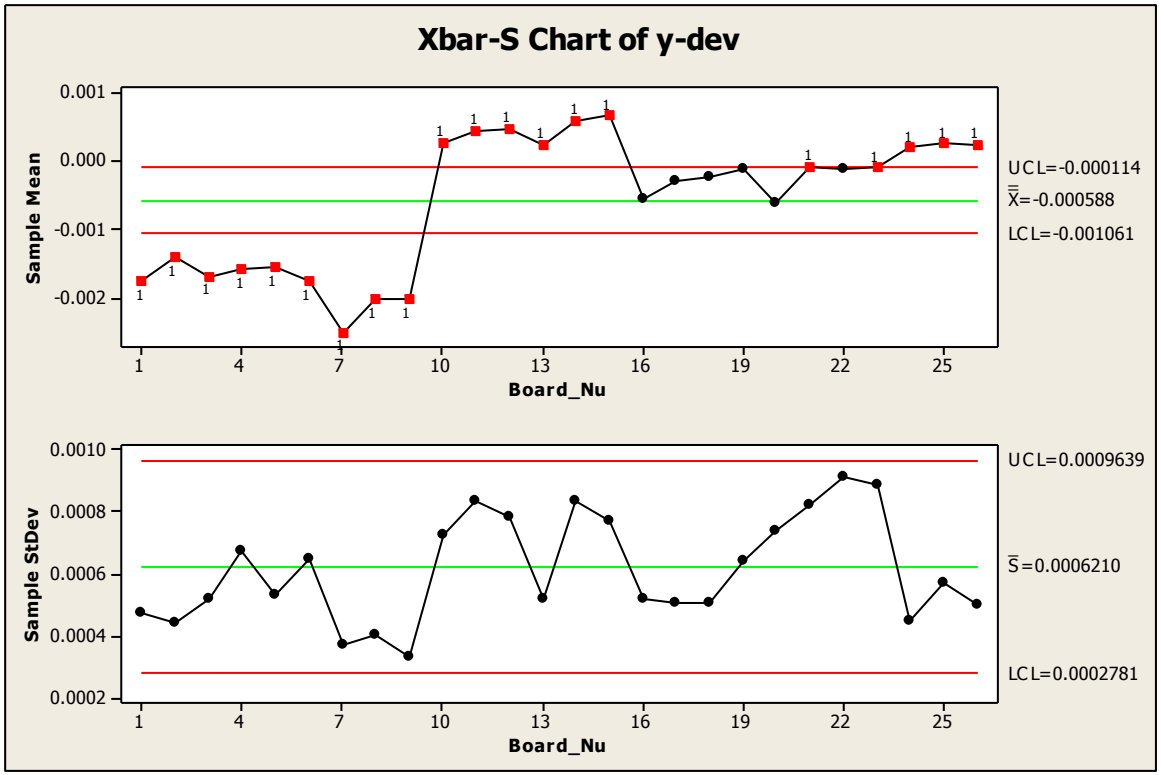


# Control Chart for x-dev

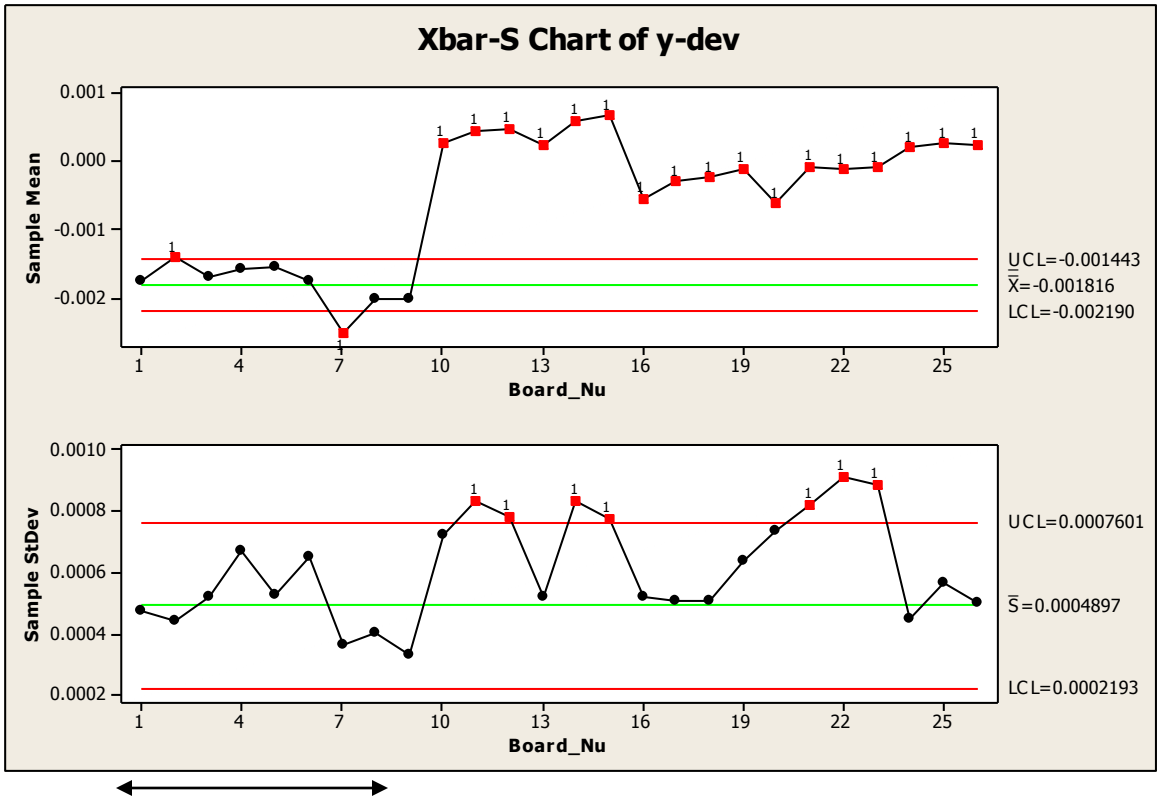



Control limits computed with group 1 only

# Control Chart for y-dev

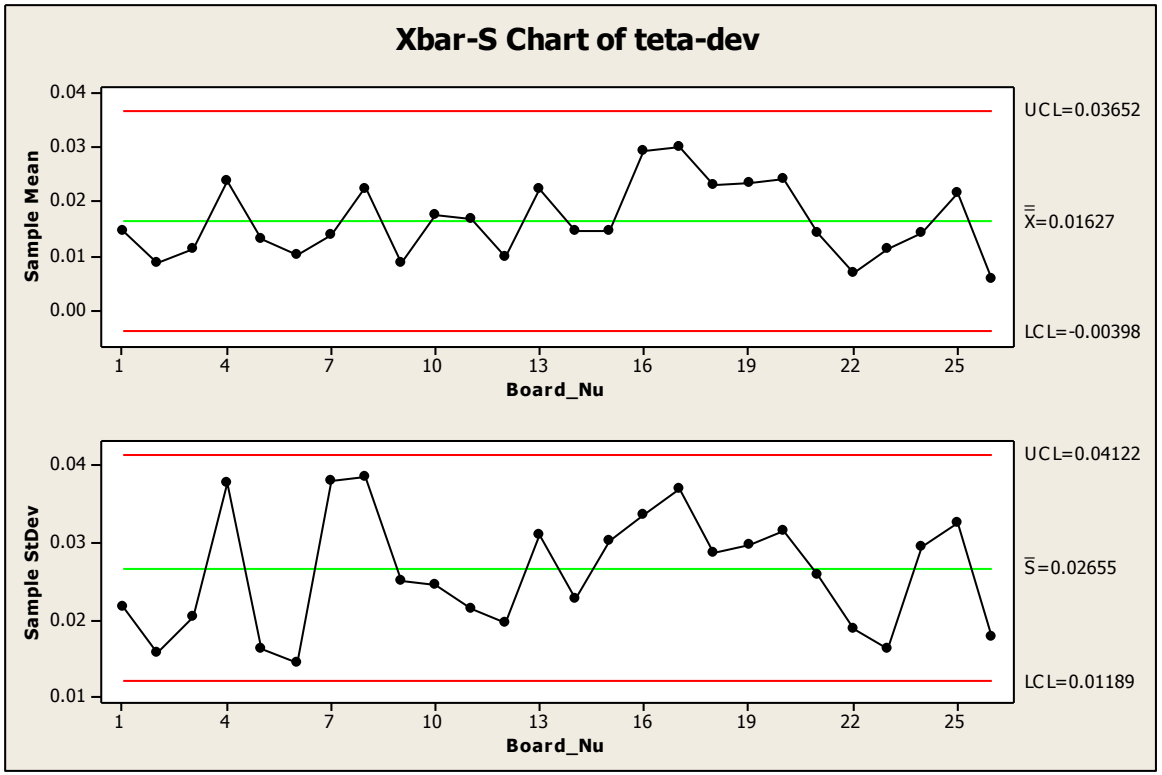


# Control Chart for y-dev

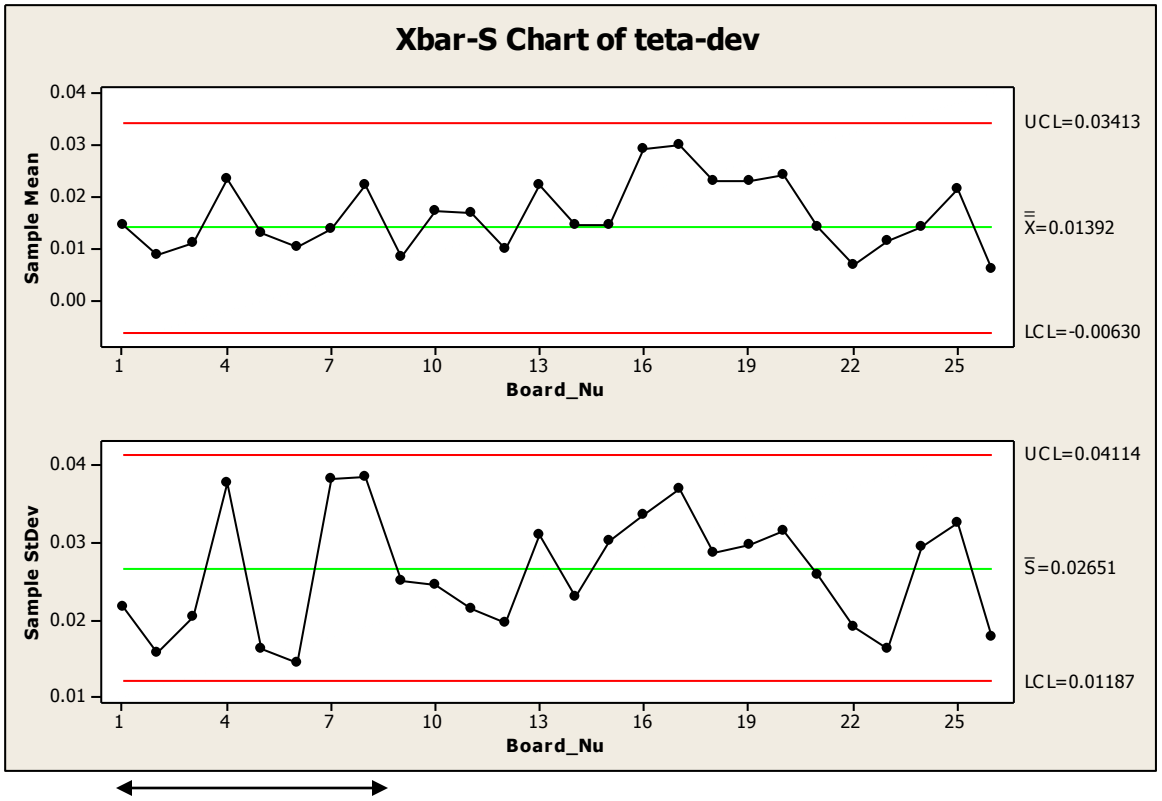



Control limits computed with group 1 only

# Control Chart for theta-dev

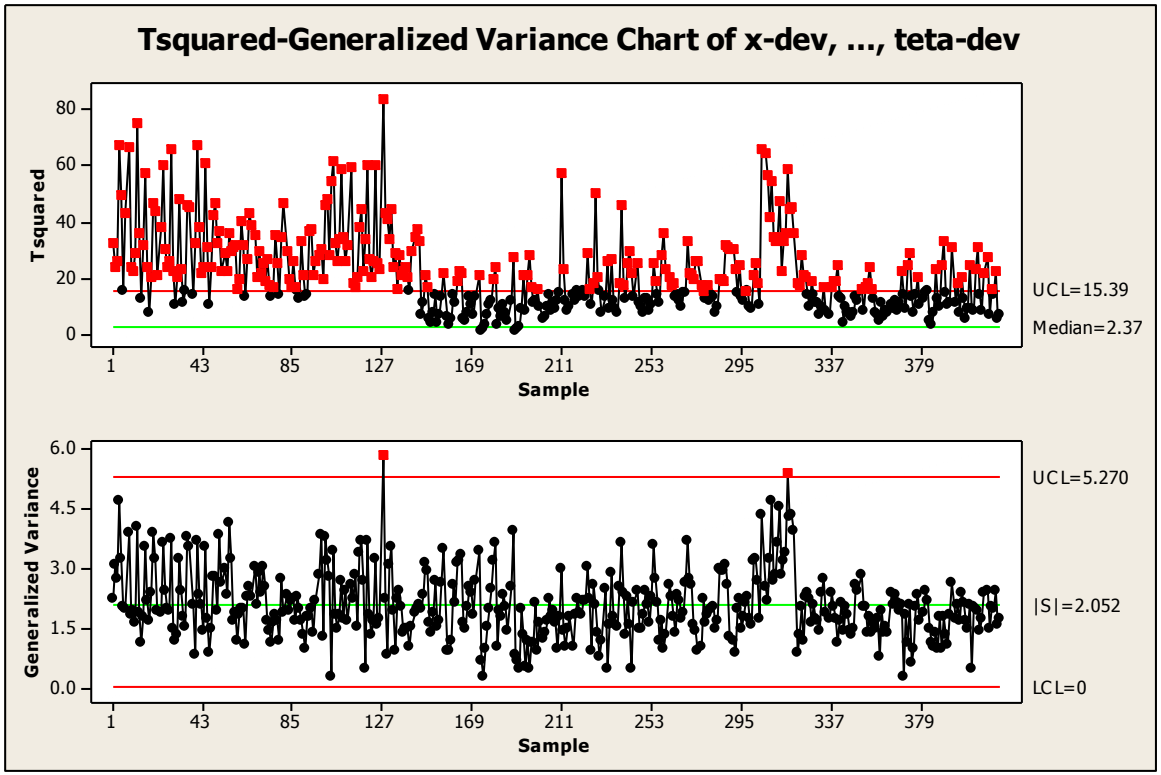


# Control Chart for theta-dev

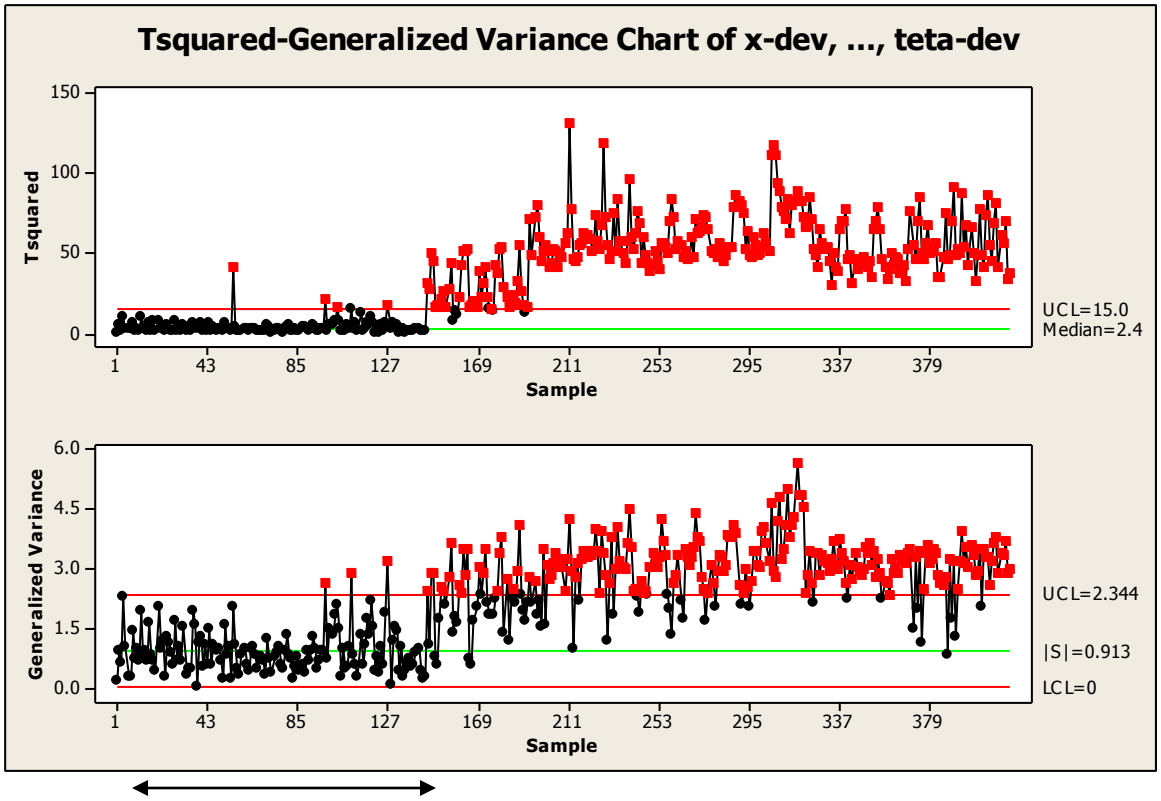



Control limits computed with group 1 only

# $T^2$ Chart


Tsquared-Generalized Variance Chart of x-dev, ..., teta-dev
Tsquared
80
60
40
20
0
1
43
85
127
169
211
253
295
337
379
Sample
UCL=15.39
Median=2.37
Generalized Variance
6.0
4.5
3.0
1.5
0.0
1
43
85
127
169
211
253
295
337
379
Sample
UCL=5.270
|S|=2.052
LCL=0

# $T^2$ Chart

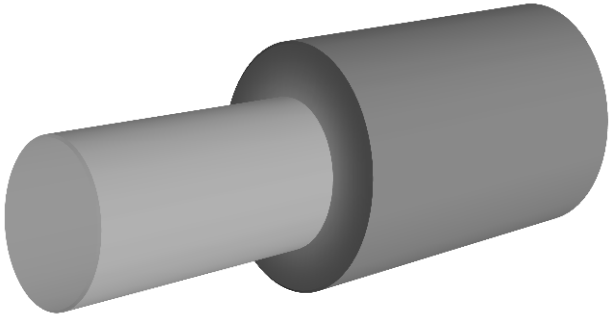



Control limits computed with group 1 only

# The Aluminum Pin Data

6 variables:

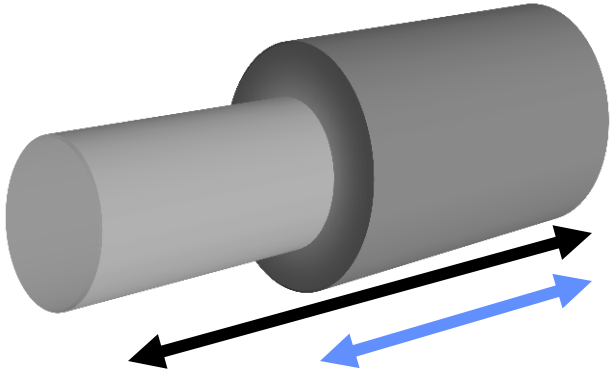
- 4 diameters
- 2 lengths



70 observations:

- 30 in phase I
- 40 in on-going control

# Multivariate Control Charts*

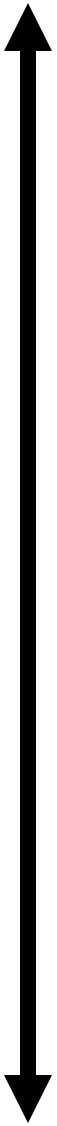


| Diameter 11 | Diameter 21 | Diameter 31 | Diameter 4 | Length 1 | Length 2 |
|---|---|---|---|---|---|
| 9.99 | 9.97 | 9.96 | 14.97 | 49.89 | 60.02 |
| 9.96 | 9.96 | 9.95 | 14.94 | 49.84 | 60.02 |
| 9.97 | 9.96 | 9.95 | 14.95 | 49.85 | 60.00 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.89 | 60.06 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.91 | 60.09 |
| 9.99 | 9.99 | 9.98 | 14.99 | 49.91 | 60.08 |
| 10.00 | 9.99 | 9.99 | 14.98 | 49.91 | 60.08 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.89 | 60.09 |
| 9.96 | 9.95 | 9.95 | 14.95 | 50.00 | 60.15 |
| 9.99 | 9.98 | 9.98 | 14.99 | 49.86 | 60.06 |
| 10.00 | 9.99 | 9.98 | 14.99 | 49.94 | 60.08 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.05 |
| 9.97 | 9.96 | 9.96 | 14.96 | 49.90 | 60.02 |
| 9.97 | 9.96 | 9.96 | 14.96 | 49.91 | 60.02 |
| 9.97 | 9.97 | 9.96 | 14.97 | 49.90 | 60.01 |

*Fuchs C. and Kenett, R., Multivariate Quality Control Theory and Applications, Dekker, 1998

# Example of two dimensional individual observations

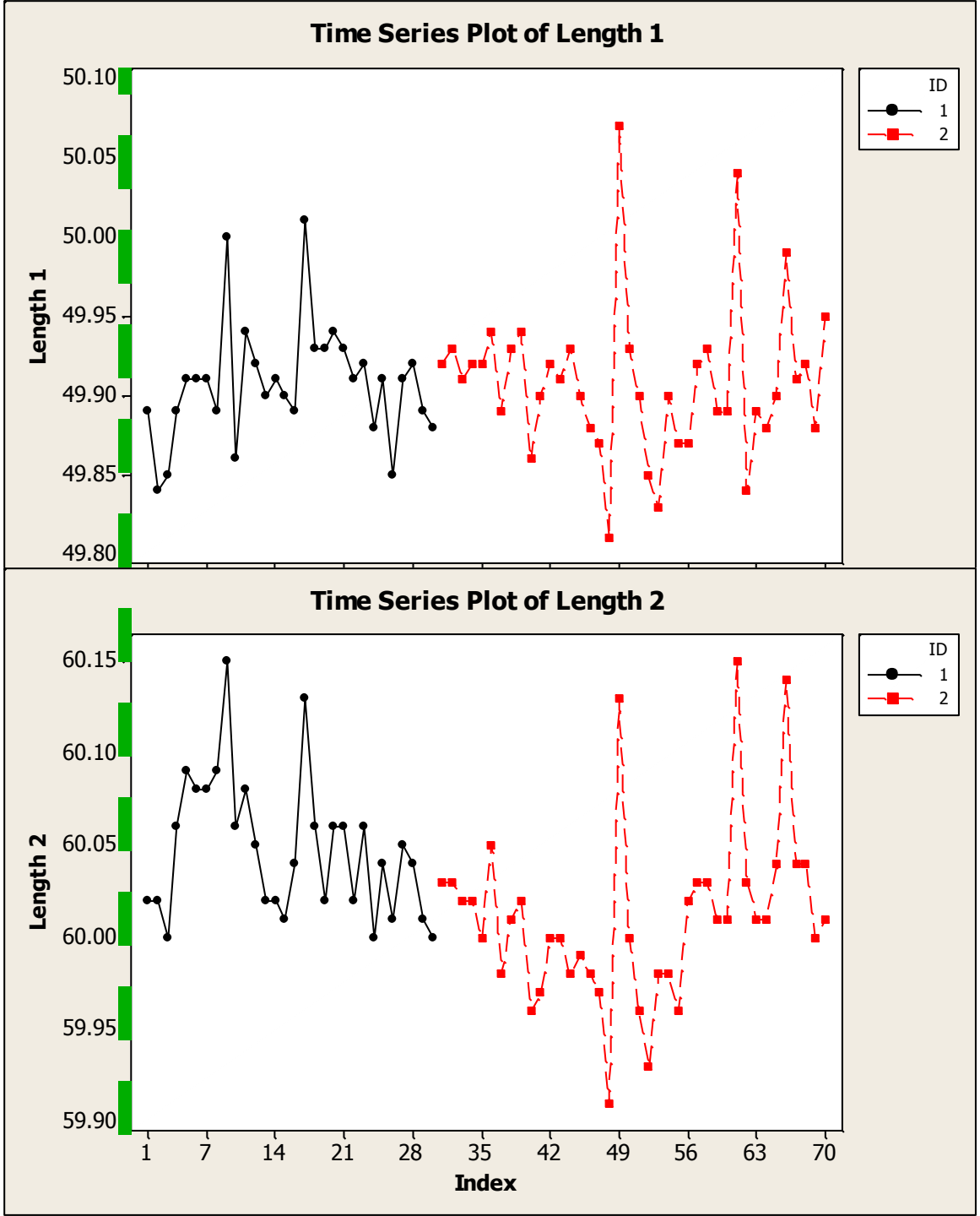
| Diameter 11 | Diameter 21 | Diameter 31 | Diameter 41 | Length 1 | Length 2 | ID |
|---|---|---|---|---|---|---|
| 9.99 | 9.97 | 9.96 | 14.97 | 49.89 | 60.02 | 1 |
| 9.96 | 9.96 | 9.95 | 14.94 | 49.84 | 60.02 | 1 |
| 9.97 | 9.96 | 9.95 | 14.95 | 49.85 | 60.00 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.89 | 60.06 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.91 | 60.09 | 1 |
| 9.99 | 9.99 | 9.98 | 14.99 | 49.91 | 60.08 | 1 |
| 10.00 | 9.99 | 9.99 | 14.98 | 49.91 | 60.08 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.89 | 60.09 | 1 |
| 9.96 | 9.95 | 9.95 | 14.95 | 50.00 | 60.15 | 1 |
| 9.99 | 9.98 | 9.98 | 14.99 | 49.86 | 60.06 | 1 |
| 10.00 | 9.99 | 9.98 | 14.99 | 49.94 | 60.08 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.05 | 1 |
| 9.97 | 9.96 | 9.96 | 14.96 | 49.90 | 60.02 | 1 |
| 9.97 | 9.96 | 9.96 | 14.96 | 49.91 | 60.02 | 1 |
| 9.97 | 9.97 | 9.96 | 14.97 | 49.90 | 60.01 | 1 |
| 9.97 | 9.97 | 9.96 | 14.97 | 49.89 | 60.04 | 1 |
| 9.98 | 9.97 | 9.96 | 14.96 | 50.01 | 60.13 | 1 |
| 9.98 | 9.97 | 9.97 | 14.96 | 49.93 | 60.06 | 1 |
| 9.98 | 9.98 | 9.97 | 14.98 | 49.93 | 60.02 | 1 |
| 9.98 | 9.97 | 9.97 | 14.97 | 49.94 | 60.06 | 1 |
| 9.98 | 9.97 | 9.97 | 14.97 | 49.93 | 60.06 | 1 |
| 9.98 | 9.97 | 9.97 | 14.97 | 49.91 | 60.02 | 1 |
| 9.98 | 9.97 | 9.96 | 14.98 | 49.92 | 60.06 | 1 |
| 10.00 | 9.99 | 9.98 | 14.98 | 49.88 | 60.00 | 1 |
| 9.99 | 9.99 | 9.99 | 14.98 | 49.91 | 60.04 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.85 | 60.01 | 1 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.91 | 60.05 | 1 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.92 | 60.04 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.89 | 60.01 | 1 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.88 | 60.00 | 1 |

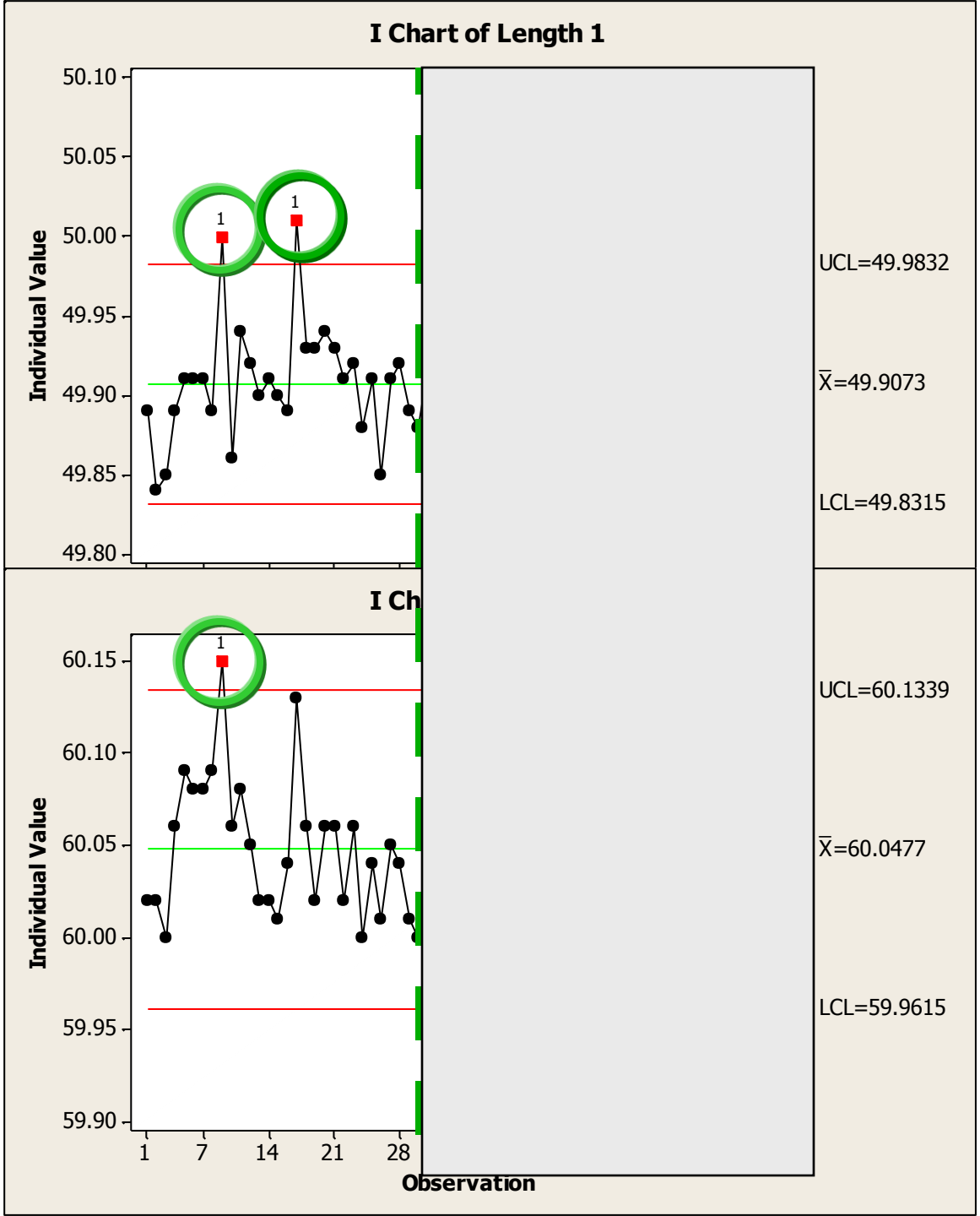


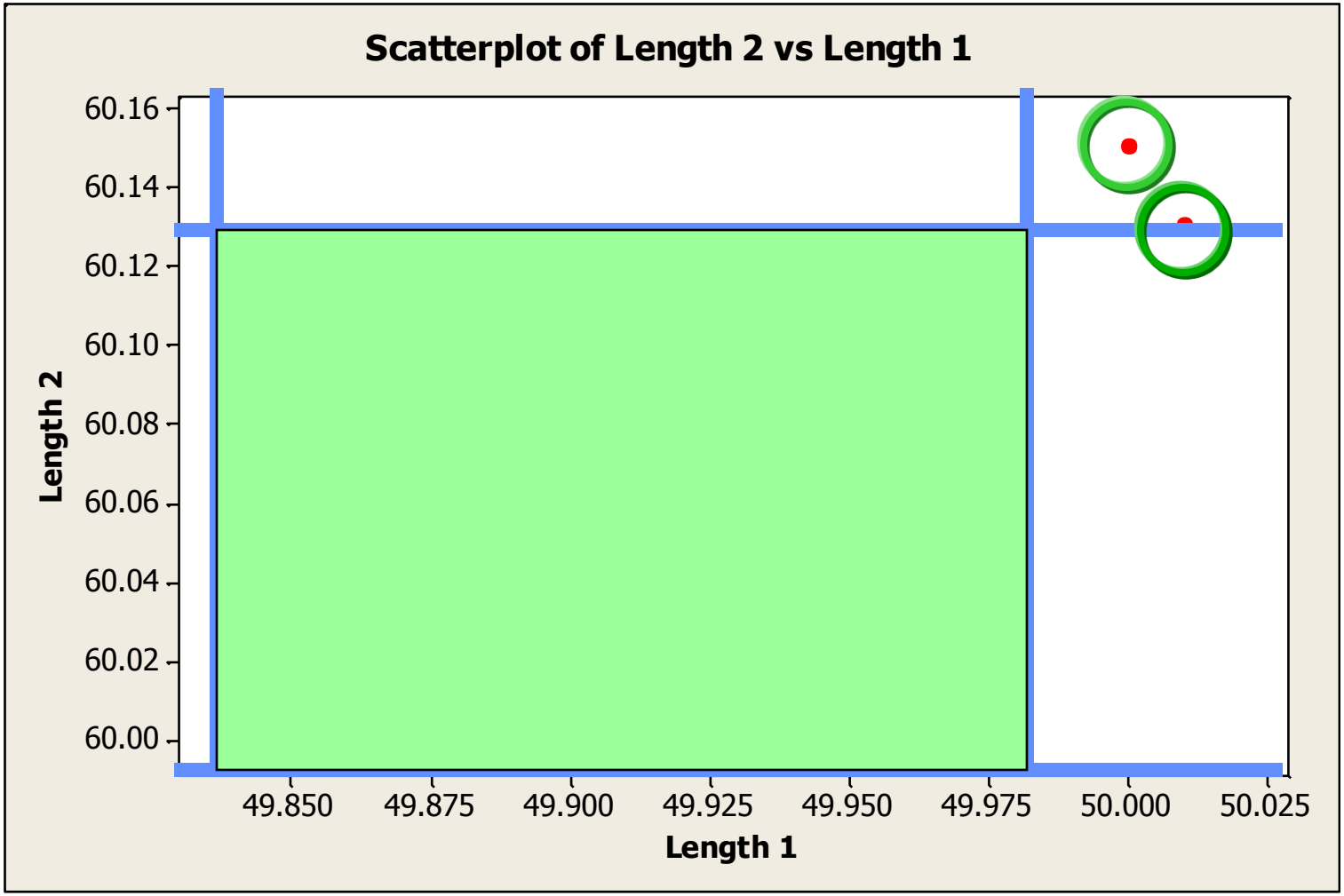
30 observations

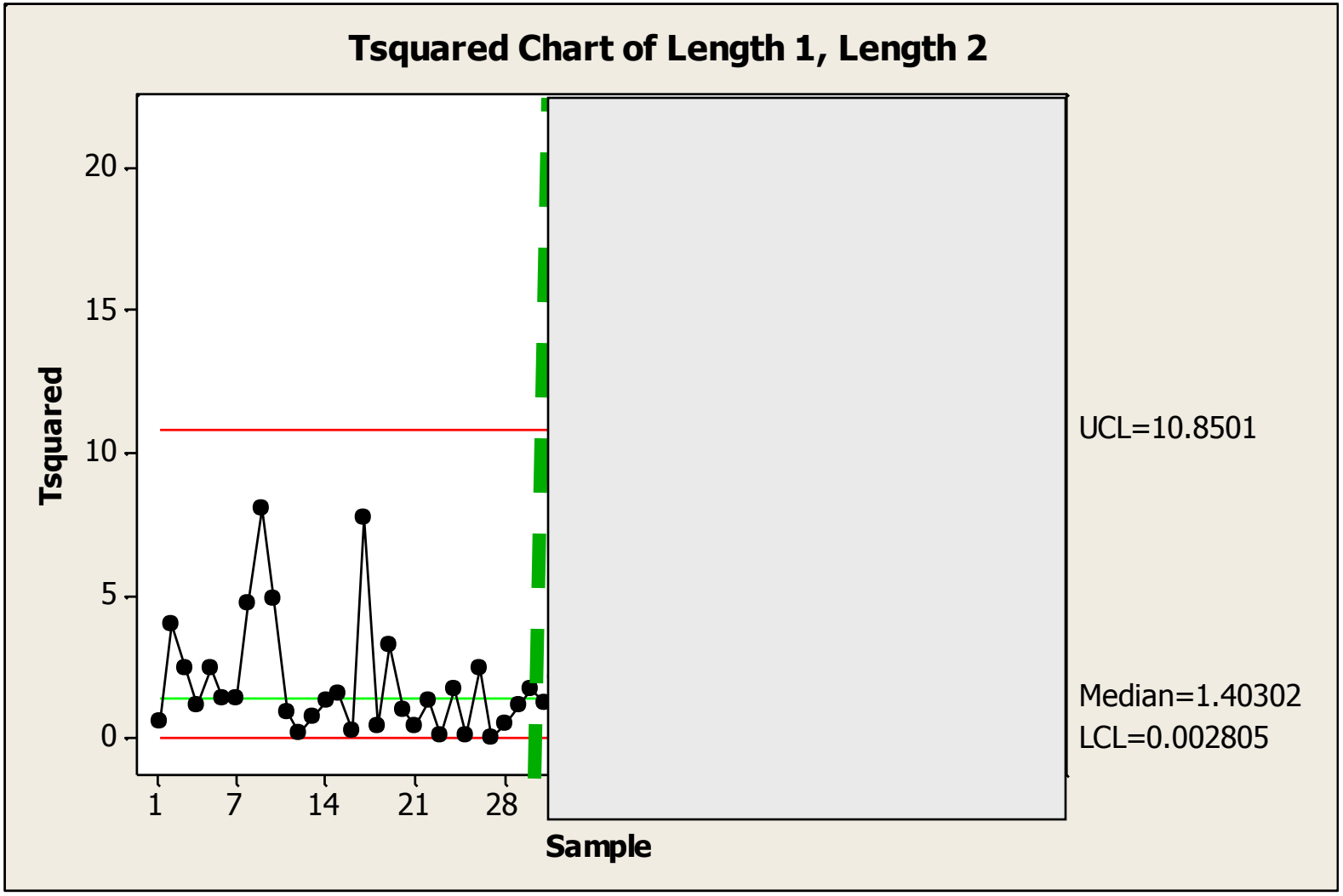
| Diameter 11 | Diameter 21 | Diameter 31 | Diameter 41 | Length 1 | Length 2 | ID |
|---|---|---|---|---|---|---|
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.03 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.93 | 60.03 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.91 | 60.02 | 2 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.02 | 2 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.00 | 2 |
| 10.00 | 10.00 | 9.99 | 15.00 | 49.94 | 60.05 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.89 | 59.98 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.93 | 60.01 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.94 | 60.02 | 2 |
| 10.00 | 10.00 | 9.99 | 15.00 | 49.86 | 59.96 | 2 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.90 | 59.97 | 2 |
| 10.00 | 10.00 | 10.00 | 14.99 | 49.92 | 60.00 | 2 |
| 10.00 | 10.00 | 9.99 | 14.98 | 49.91 | 60.00 | 2 |
| 10.00 | 10.00 | 10.00 | 15.00 | 49.93 | 59.98 | 2 |
| 10.00 | 9.99 | 9.98 | 14.98 | 49.90 | 59.99 | 2 |
| 9.99 | 9.99 | 9.99 | 14.99 | 49.88 | 59.98 | 2 |
| 10.01 | 10.01 | 10.01 | 15.01 | 49.87 | 59.97 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.81 | 59.91 | 2 |
| 10.01 | 10.00 | 10.00 | 15.01 | 50.07 | 60.13 | 2 |
| 10.01 | 10.00 | 10.00 | 15.00 | 49.93 | 60.00 | 2 |
| 10.00 | 10.00 | 10.00 | 14.99 | 49.90 | 59.96 | 2 |
| 10.01 | 10.01 | 10.01 | 15.00 | 49.85 | 59.93 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.83 | 59.98 | 2 |
| 10.01 | 10.01 | 10.00 | 14.99 | 49.90 | 59.98 | 2 |
| 10.01 | 10.01 | 10.00 | 15.00 | 49.87 | 59.96 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.87 | 60.02 | 2 |
| 9.99 | 9.99 | 9.99 | 14.98 | 49.92 | 60.03 | 2 |
| 9.99 | 9.98 | 9.98 | 14.99 | 49.93 | 60.03 | 2 |
| 9.99 | 9.99 | 9.98 | 14.99 | 49.89 | 60.01 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.89 | 60.01 | 2 |
| 9.99 | 9.99 | 9.99 | 15.00 | 50.04 | 60.15 | 2 |
| 10.00 | 10.00 | 10.00 | 14.99 | 49.84 | 60.03 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.89 | 60.01 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.88 | 60.01 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.90 | 60.04 | 2 |
| 9.90 | 9.89 | 9.91 | 14.88 | 49.99 | 60.14 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.91 | 60.04 | 2 |
| 9.99 | 9.99 | 9.99 | 14.98 | 49.92 | 60.04 | 2 |
| 10.01 | 10.01 | 10.00 | 15.00 | 49.88 | 60.00 | 2 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.95 | 60.01 | 2 |

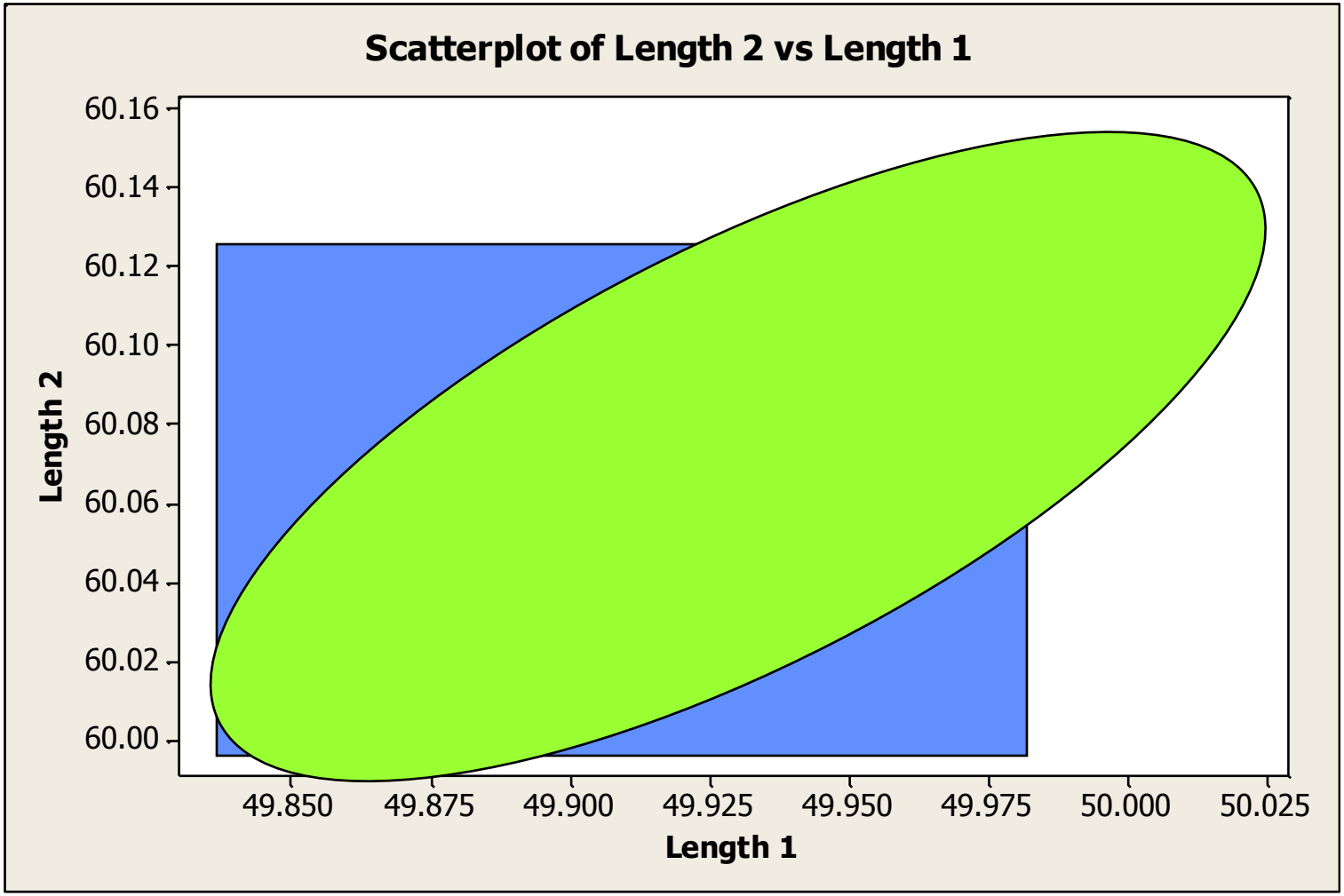
40 observations in on-going control

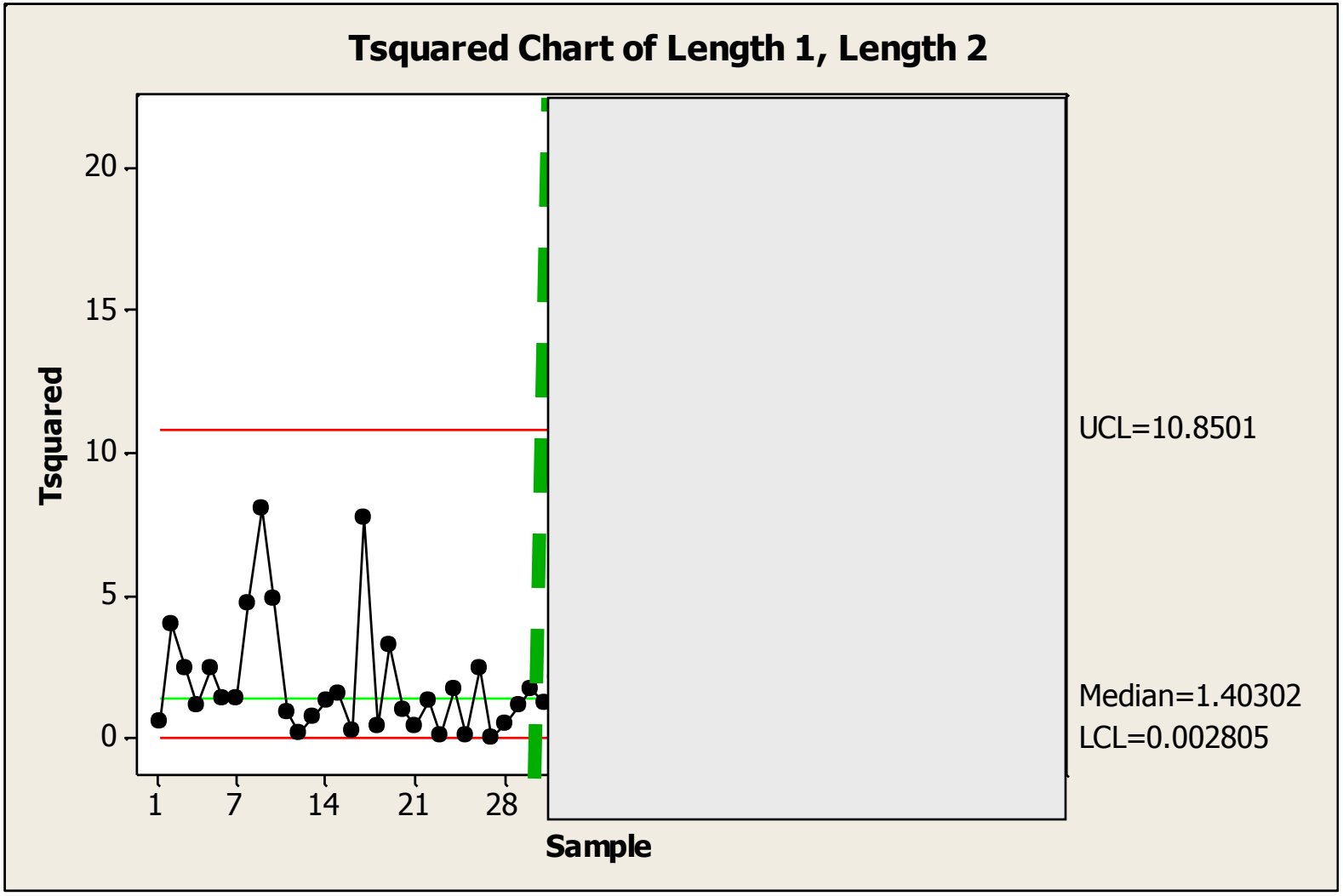

Time Series Plot of Length 1
Length 1
50.10
50.05
50.00
49.95
49.90
49.85
49.80
ID
1
2
Time Series Plot of Length 2
Length 2
60.15
60.10
60.05
60.00
59.95
59.90
1
7
14
21
28
35
42
49
56
63
70
Index
ID
1
2

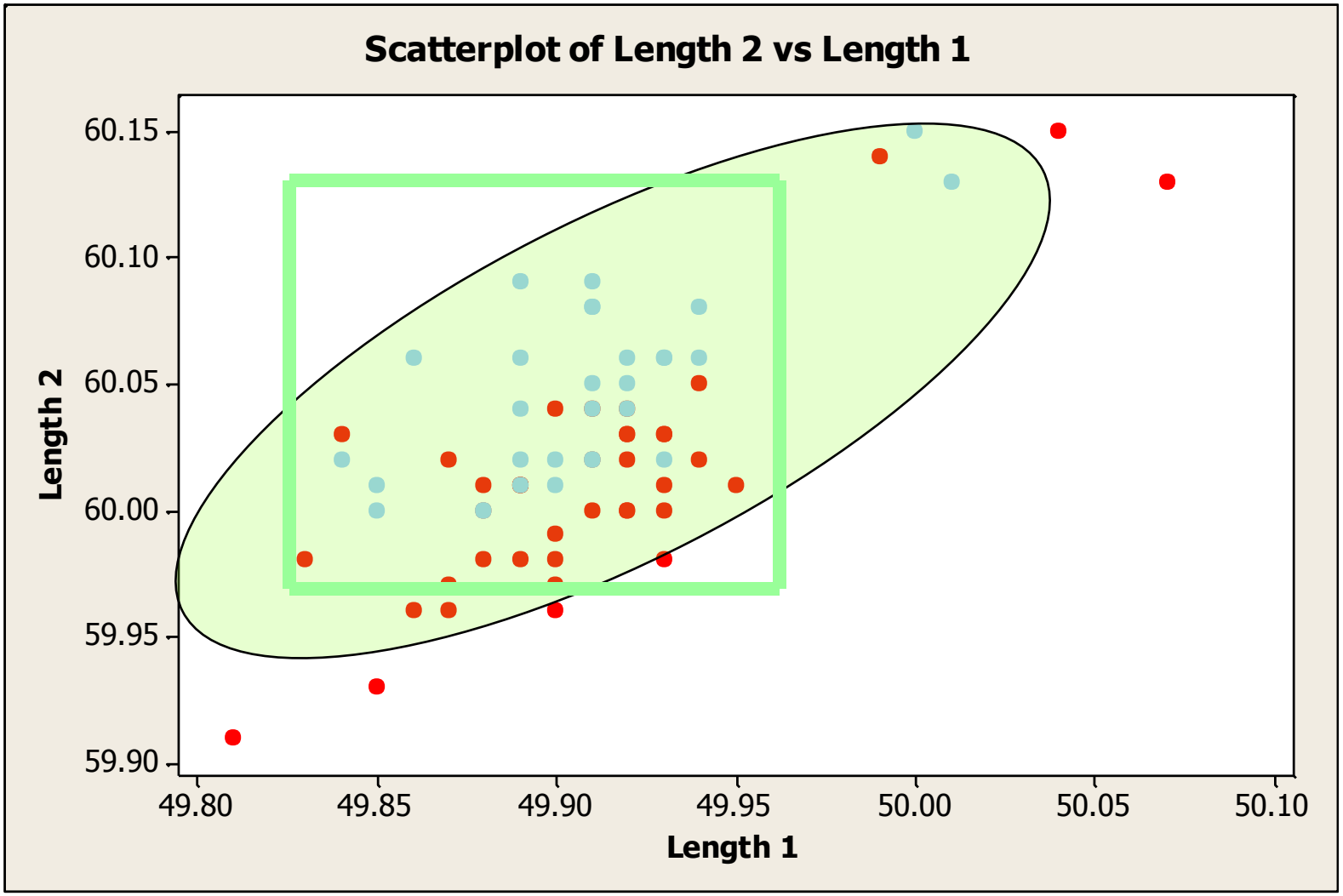

I Chart of Length 1
Individual Value
50.10
50.05
50.00
49.95
49.90
49.85
49.80
1
1
UCL=49.9832
X̄=49.9073
LCL=49.8315
I Ch
Individual Value
60.15
60.10
60.05
60.00
59.95
59.90
1
UCL=60.1339
X̄=60.0477
LCL=59.9615
1
7
14
21
28
Observation

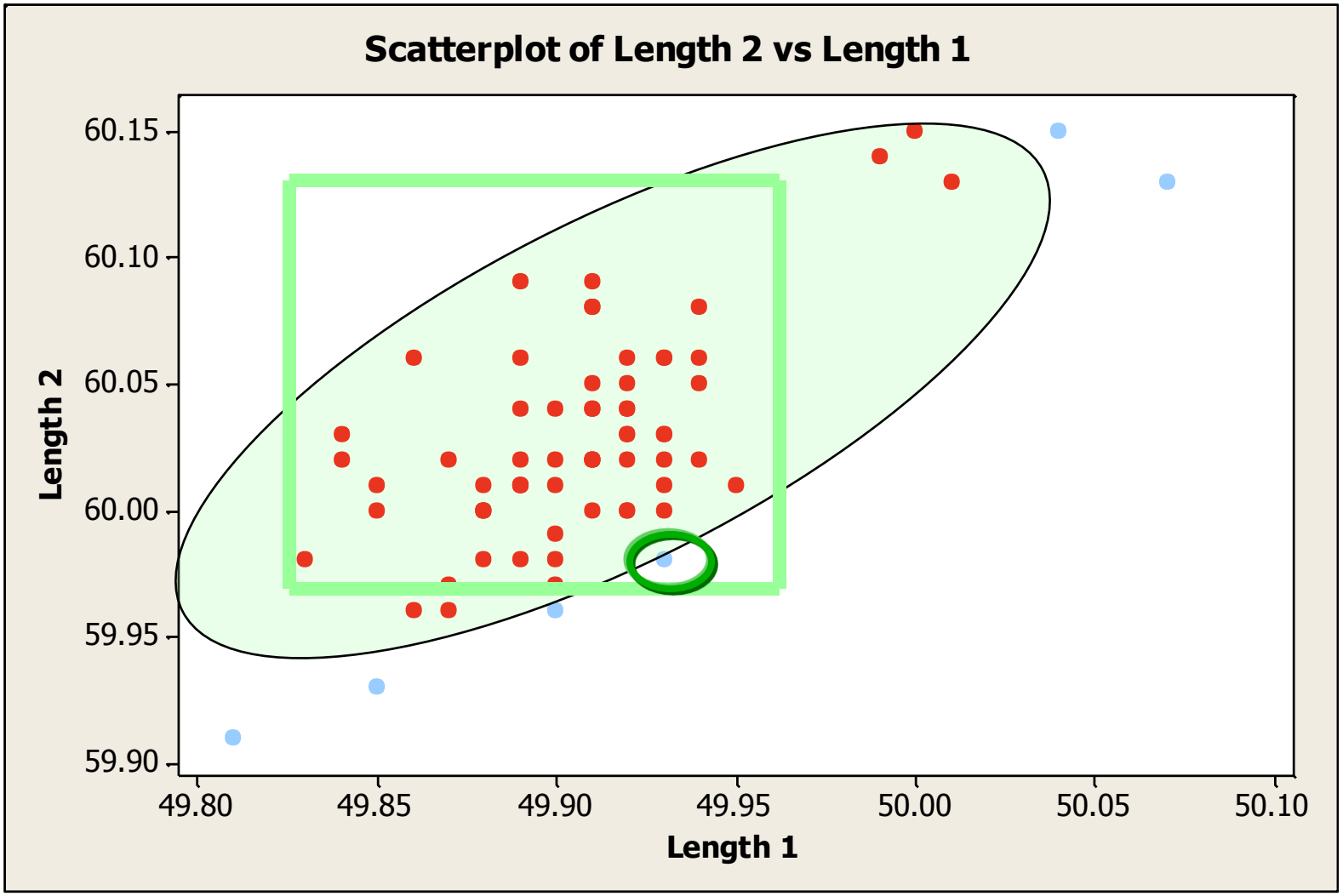

Scatterplot of Length 2 vs Length 1
Length 2
60.16
60.14
60.12
60.10
60.08
60.06
60.04
60.02
60.00
49.850
49.875
49.900
49.925
49.950
49.975
50.000
50.025
Length 1

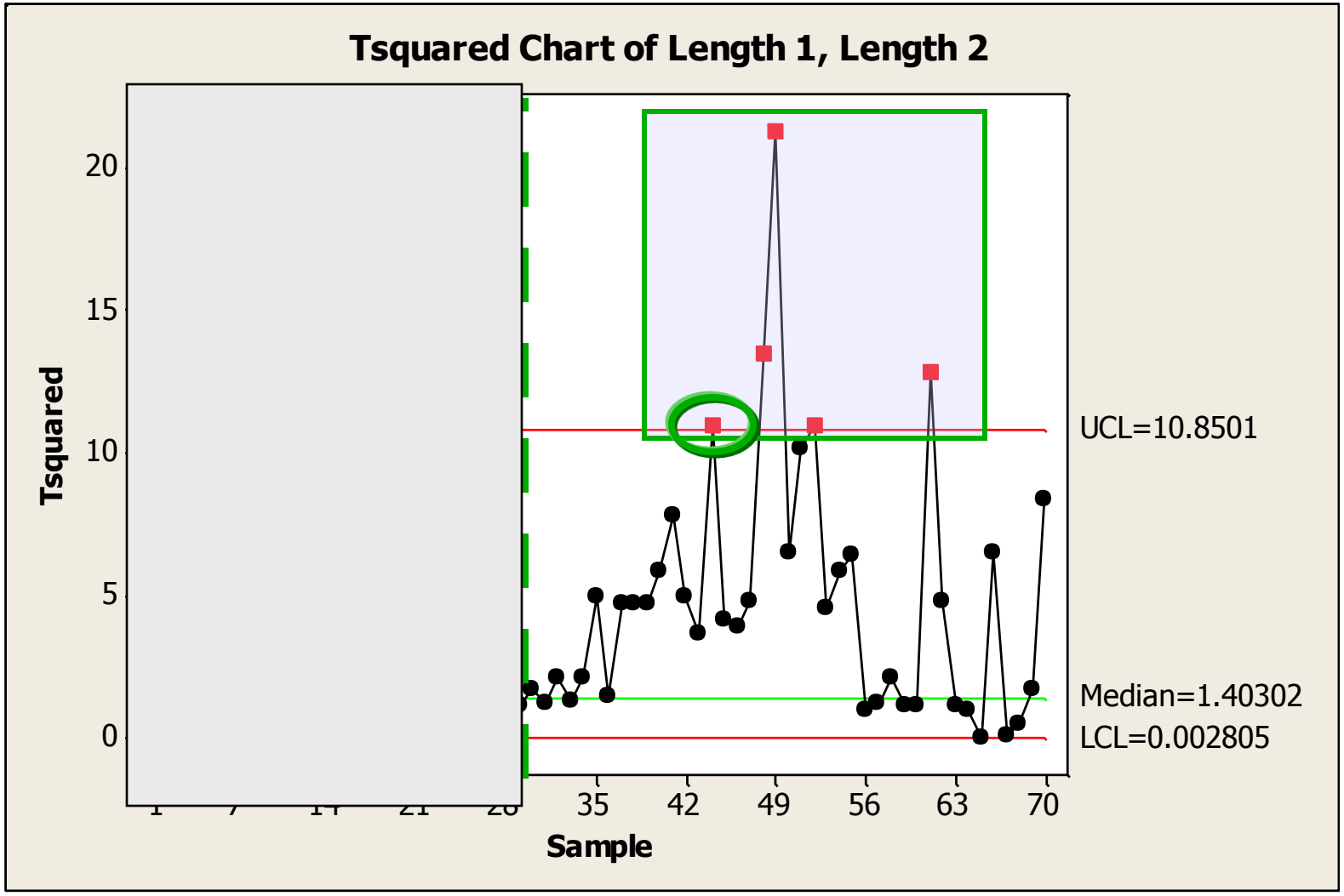

Tsquared Chart of Length 1, Length 2
Tsquared
20
15
10
5
0
1
7
14
21
28
Sample
UCL=10.8501
Median=1.40302
LCL=0.002805

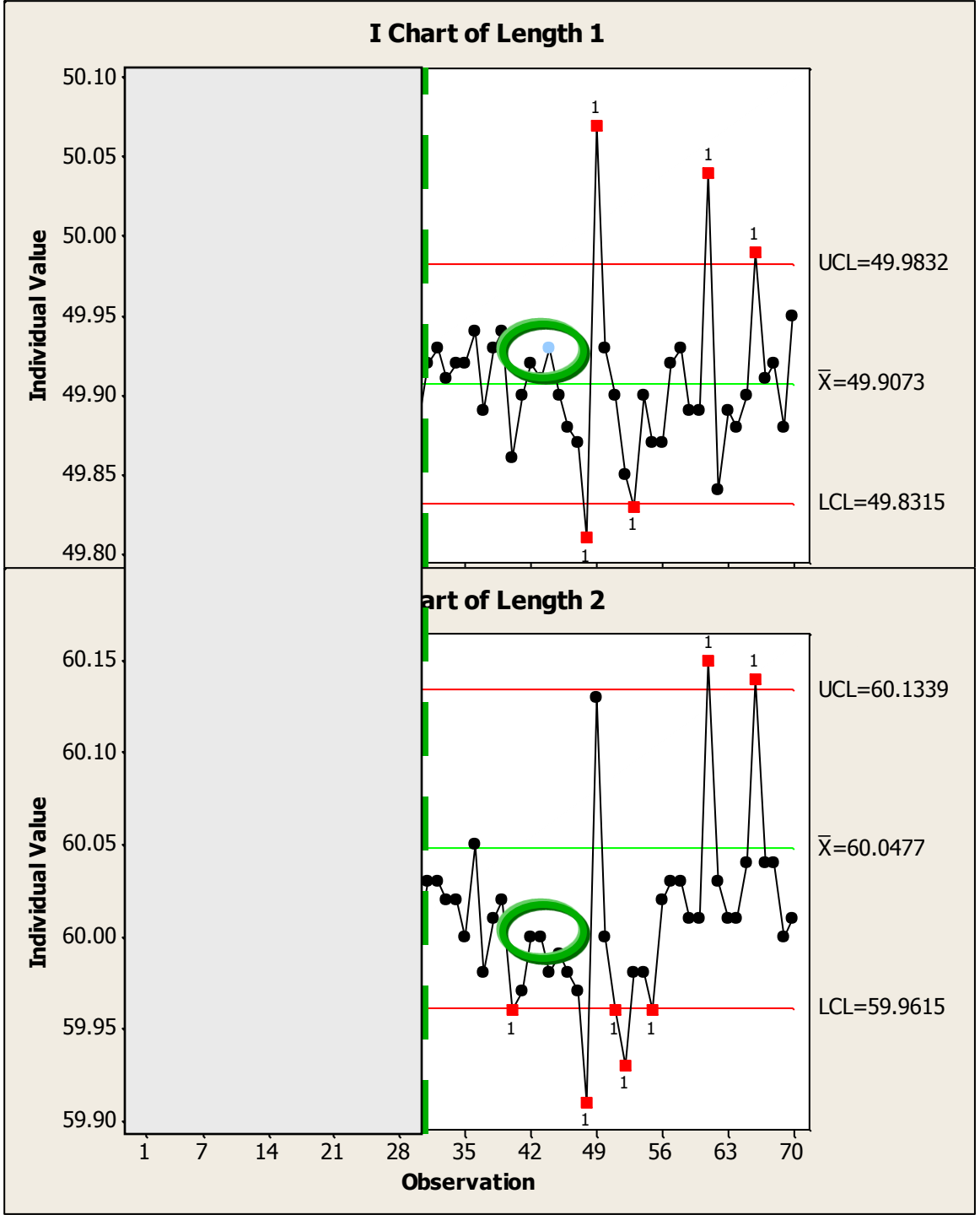

Scatterplot of Length 2 vs Length 1
Length 2
60.16
60.14
60.12
60.10
60.08
60.06
60.04
60.02
60.00
49.850
49.875
49.900
49.925
49.950
49.975
50.000
50.025
Length 1


Tsquared Chart of Length 1, Length 2
Tsquared
20
15
10
5
0
1
7
14
21
28
Sample
UCL=10.8501
Median=1.40302
LCL=0.002805


Scatterplot of Length 2 vs Length 1
Length 2
60.15
60.10
60.05
60.00
59.95
59.90
49.80
49.85
49.90
49.95
50.00
50.05
50.10
Length 1

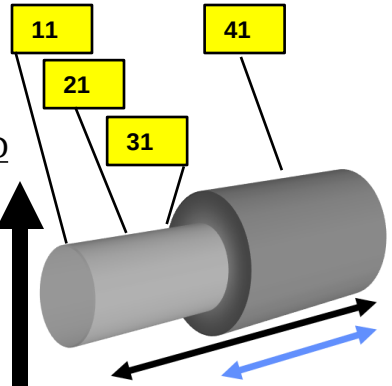

Scatterplot of Length 2 vs Length 1
Length 2
60.15
60.10
60.05
60.00
59.95
59.90
49.80
49.85
49.90
49.95
50.00
50.05
50.10
Length 1


Tsquared Chart of Length 1, Length 2
Tsquared
20
15
10
5
0
35
42
49
56
63
70
Sample
UCL=10.8501
Median=1.40302
LCL=0.002805

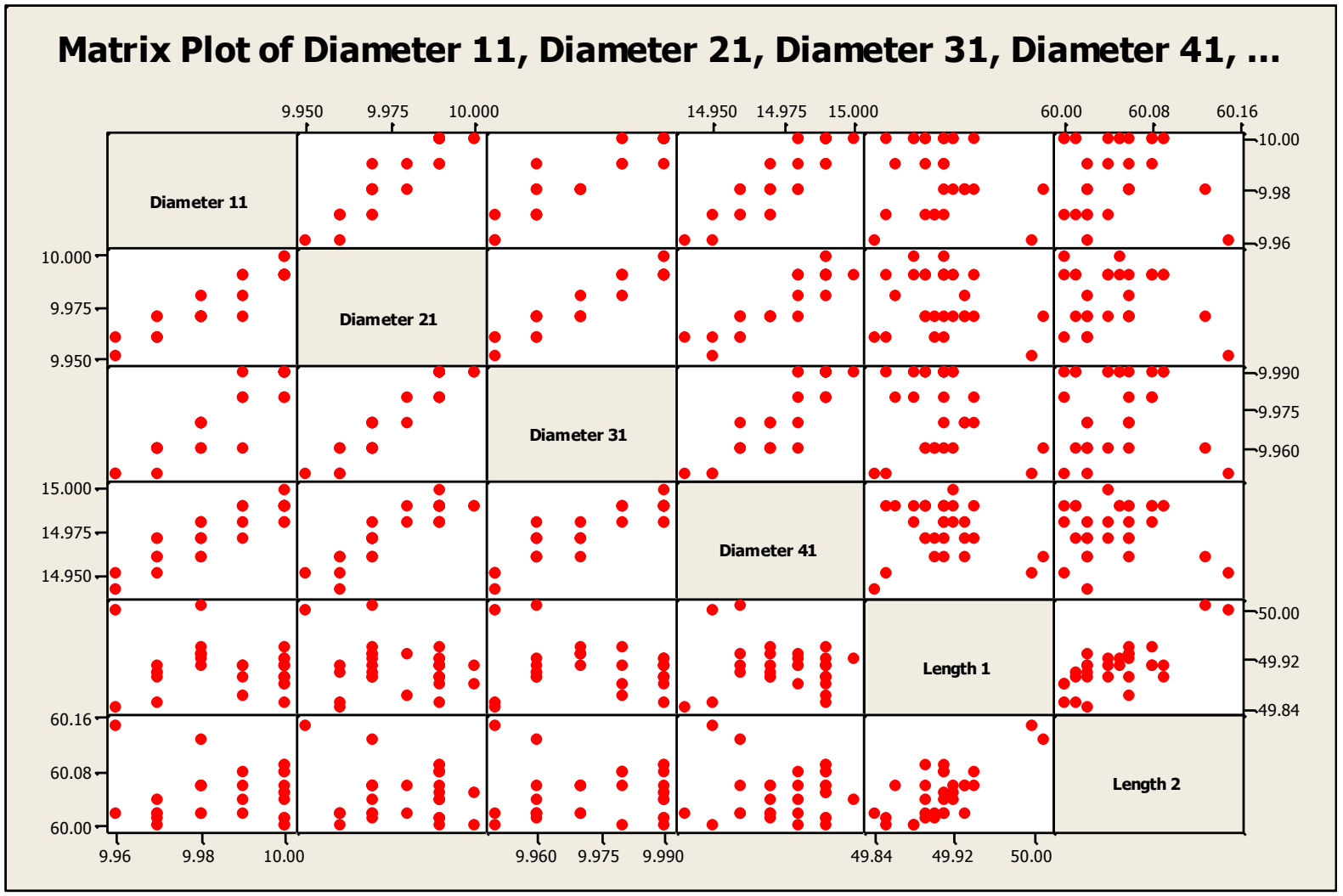

I Chart of Length 1
Individual Value
50.10
50.05
50.00
49.95
49.90
49.85
49.80
UCL=49.9832
X̄=49.9073
LCL=49.8315
art of Length 2
Individual Value
60.15
60.10
60.05
60.00
59.95
59.90
UCL=60.1339
X̄=60.0477
LCL=59.9615
1 7 14 21 28 35 42 49 56 63 70
Observation

# Now with six variables

| Diameter 11 | Diameter 21 | Diameter 31 | Diameter 41 | Length 1 | Length 2 | ID |
|---|---|---|---|---|---|---|
| 9.99 | 9.97 | 9.96 | 14.97 | 49.89 | 60.02 | 1 |
| 9.96 | 9.96 | 9.95 | 14.94 | 49.84 | 60.02 | 1 |
| 9.97 | 9.96 | 9.95 | 14.95 | 49.85 | 60.00 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.89 | 60.06 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.91 | 60.09 | 1 |
| 9.99 | 9.99 | 9.98 | 14.99 | 49.91 | 60.08 | 1 |
| 10.00 | 9.99 | 9.99 | 14.98 | 49.91 | 60.08 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.89 | 60.09 | 1 |
| 9.96 | 9.95 | 9.95 | 14.95 | 50.00 | 60.15 | 1 |
| 9.99 | 9.98 | 9.98 | 14.99 | 49.86 | 60.06 | 1 |
| 10.00 | 9.99 | 9.98 | 14.99 | 49.94 | 60.08 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.05 | 1 |
| 9.97 | 9.96 | 9.96 | 14.96 | 49.90 | 60.02 | 1 |
| 9.97 | 9.96 | 9.96 | 14.96 | 49.91 | 60.02 | 1 |
| 9.97 | 9.97 | 9.96 | 14.97 | 49.90 | 60.01 | 1 |
| 9.97 | 9.97 | 9.96 | 14.97 | 49.89 | 60.04 | 1 |
| 9.98 | 9.97 | 9.96 | 14.96 | 50.01 | 60.13 | 1 |
| 9.98 | 9.97 | 9.97 | 14.96 | 49.93 | 60.06 | 1 |
| 9.98 | 9.98 | 9.97 | 14.98 | 49.93 | 60.02 | 1 |
| 9.98 | 9.97 | 9.97 | 14.97 | 49.94 | 60.06 | 1 |
| 9.98 | 9.97 | 9.97 | 14.97 | 49.93 | 60.06 | 1 |
| 9.98 | 9.97 | 9.97 | 14.97 | 49.91 | 60.02 | 1 |
| 9.98 | 9.97 | 9.96 | 14.98 | 49.92 | 60.06 | 1 |
| 10.00 | 9.99 | 9.98 | 14.98 | 49.88 | 60.00 | 1 |
| 9.99 | 9.99 | 9.99 | 14.98 | 49.91 | 60.04 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.85 | 60.01 | 1 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.91 | 60.05 | 1 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.92 | 60.04 | 1 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.89 | 60.01 | 1 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.88 | 60.00 | 1 |

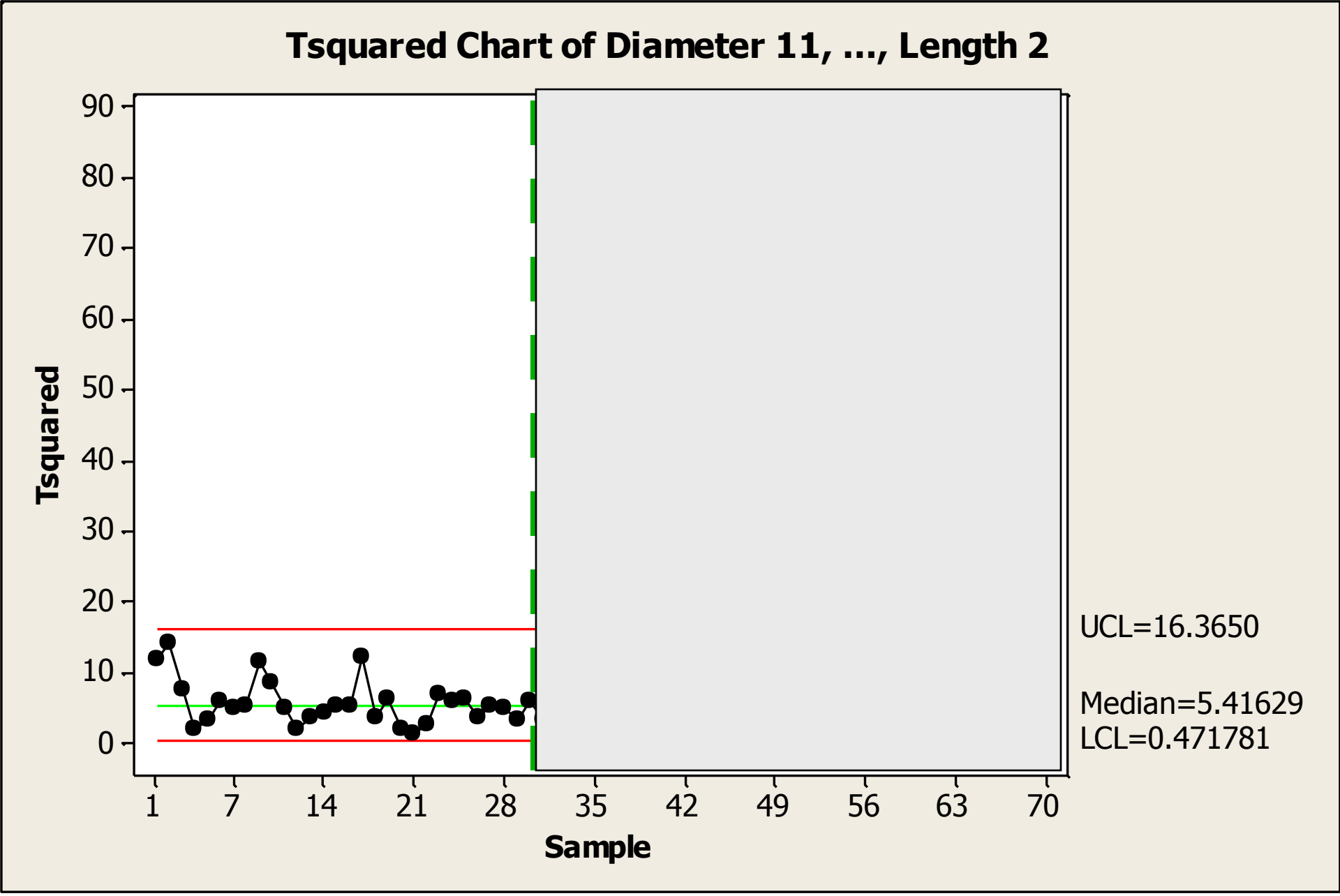



30 observations


Matrix Plot of Diameter 11, Diameter 21, Diameter 31, Diameter 41, ...
Diameter 11
Diameter 21
Diameter 31
Diameter 41
Length 1
Length 2
9.950
9.975
10.000
14.950
14.975
15.000
60.00
60.08
60.16
10.00
9.98
9.96
9.990
9.975
9.960
50.00
49.92
49.84
9.96
9.98
10.00
9.960
9.975
9.990
49.84
49.92
50.00

| Diameter 11 | Diameter 21 | Diameter 31 | Diameter 41 | Length 1 | Length 2 | ID |
|---|---|---|---|---|---|---|
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.03 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.93 | 60.03 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.91 | 60.02 | 2 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.02 | 2 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.92 | 60.00 | 2 |
| 10.00 | 10.00 | 9.99 | 15.00 | 49.94 | 60.05 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.89 | 59.98 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.93 | 60.01 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.94 | 60.02 | 2 |
| 10.00 | 10.00 | 9.99 | 15.00 | 49.86 | 59.96 | 2 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.90 | 59.97 | 2 |
| 10.00 | 10.00 | 10.00 | 14.99 | 49.92 | 60.00 | 2 |
| 10.00 | 10.00 | 9.99 | 14.98 | 49.91 | 60.00 | 2 |
| 10.00 | 10.00 | 10.00 | 15.00 | 49.93 | 59.98 | 2 |
| 10.00 | 9.99 | 9.98 | 14.98 | 49.90 | 59.99 | 2 |
| 9.99 | 9.99 | 9.99 | 14.99 | 49.88 | 59.98 | 2 |
| 10.01 | 10.01 | 10.01 | 15.01 | 49.87 | 59.97 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.81 | 59.91 | 2 |
| 10.01 | 10.00 | 10.00 | 15.01 | 50.07 | 60.13 | 2 |
| 10.01 | 10.00 | 10.00 | 15.00 | 49.93 | 60.00 | 2 |
| 10.00 | 10.00 | 10.00 | 14.99 | 49.90 | 59.96 | 2 |
| 10.01 | 10.01 | 10.01 | 15.00 | 49.85 | 59.93 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.83 | 59.98 | 2 |
| 10.01 | 10.01 | 10.00 | 14.99 | 49.90 | 59.98 | 2 |
| 10.01 | 10.01 | 10.00 | 15.00 | 49.87 | 59.96 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.87 | 60.02 | 2 |
| 9.99 | 9.99 | 9.99 | 14.98 | 49.92 | 60.03 | 2 |
| 9.99 | 9.98 | 9.98 | 14.99 | 49.93 | 60.03 | 2 |
| 9.99 | 9.99 | 9.98 | 14.99 | 49.89 | 60.01 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.89 | 60.01 | 2 |
| 9.99 | 9.99 | 9.99 | 15.00 | 50.04 | 60.15 | 2 |
| 10.00 | 10.00 | 10.00 | 14.99 | 49.84 | 60.03 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.89 | 60.01 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.88 | 60.01 | 2 |
| 10.00 | 10.00 | 9.99 | 14.99 | 49.90 | 60.04 | 2 |
| 9.90 | 9.89 | 9.91 | 14.88 | 49.99 | 60.14 | 2 |
| 10.00 | 9.99 | 9.99 | 15.00 | 49.91 | 60.04 | 2 |
| 9.99 | 9.99 | 9.99 | 14.98 | 49.92 | 60.04 | 2 |
| 10.01 | 10.01 | 10.00 | 15.00 | 49.88 | 60.00 | 2 |
| 10.00 | 9.99 | 9.99 | 14.99 | 49.95 | 60.01 | 2 |

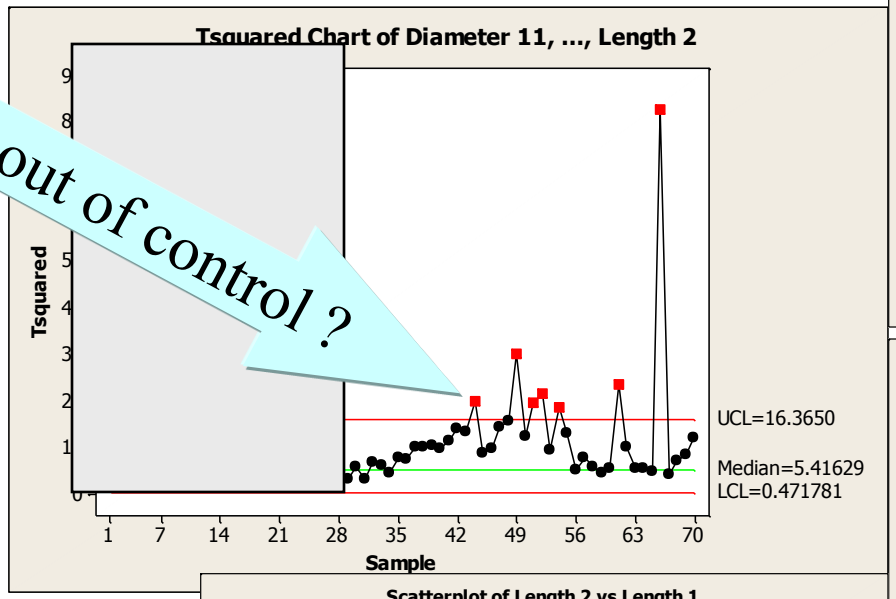
40 observations in on-going control

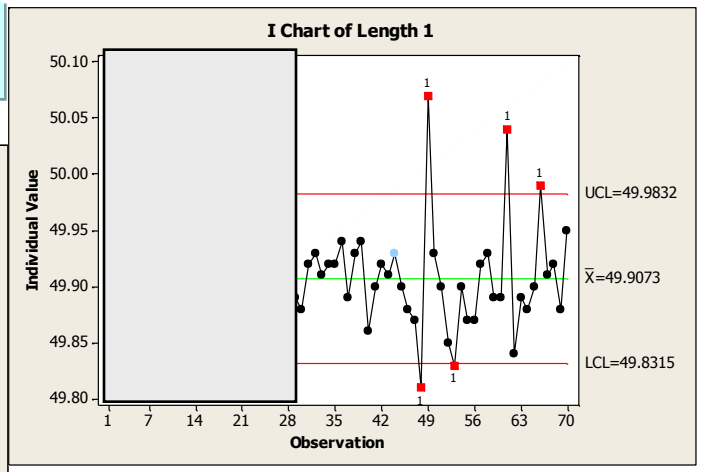

Tsquared Chart of Diameter 11, ..., Length 2
Tsquared
90
80
70
60
50
40
30
20
10
0
1
7
14
21
28
35
42
49
56
63
70
Sample
UCL=16.3650
Median=5.41629
LCL=0.471781

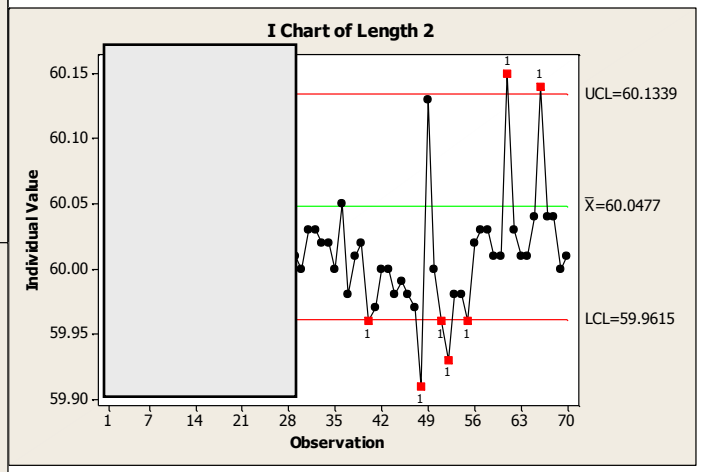
- Collect Data
- Compute $T^2$

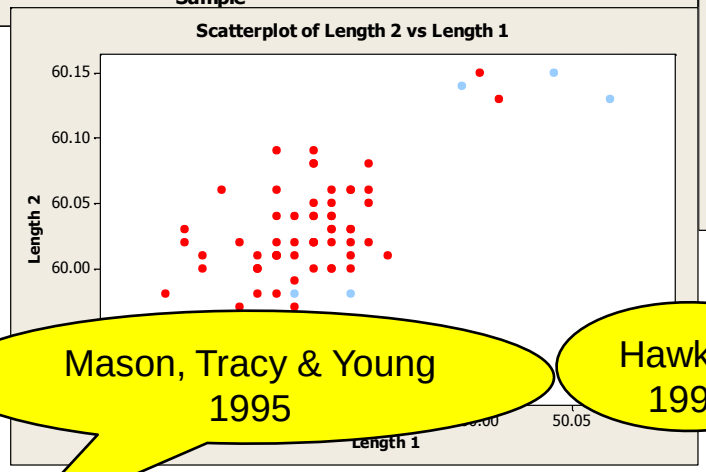
What is the special cause ?

Is process out of control ?




Roy 1958

**Step Down Procedures**
*assumes a priori ordering of variables subsets*

Mason, Tracy & Young 1995

Hawkins 1991

- Draw univariate charts
- Use graphic tools
- Use diagnostic tools

**$T^2$ Decomposition Procedures**
*assumes no a priori ordering of variables subsets*

**Regression adjusted-variables Procedures**

# Agenda

- Visualizing Multivariate Data
  - » Scatter plots
  - » Bubble plots
- Multivariate Process Control
  - » $T^2$ charts
  - » Two examples
- **Multivariate Data Analysis**
  - » **Association rules**
  - » **The Italian case study**

**Advanced**

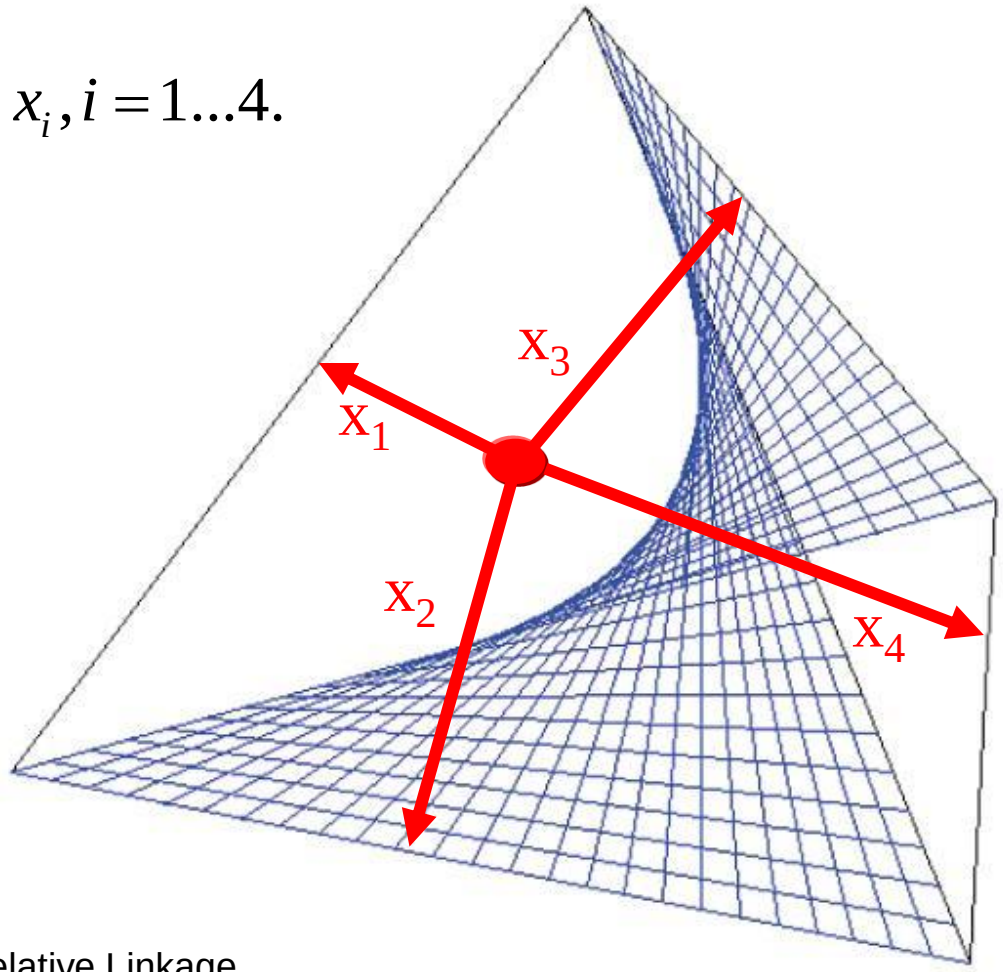
# Association Rules



| | RHS | ^RHS | |
|---|---|---|---|
| LHS | $x_1$ | $x_2$ | *g* |
| ^LHS | $x_3$ | $x_4$ | *1-g* |
| | *f* | *1-f* | 1 |


# The Simplex Representation

$$\sum_{i=1}^{4} x_i = 1,\ 0 \le x_i, i = 1...4.$$

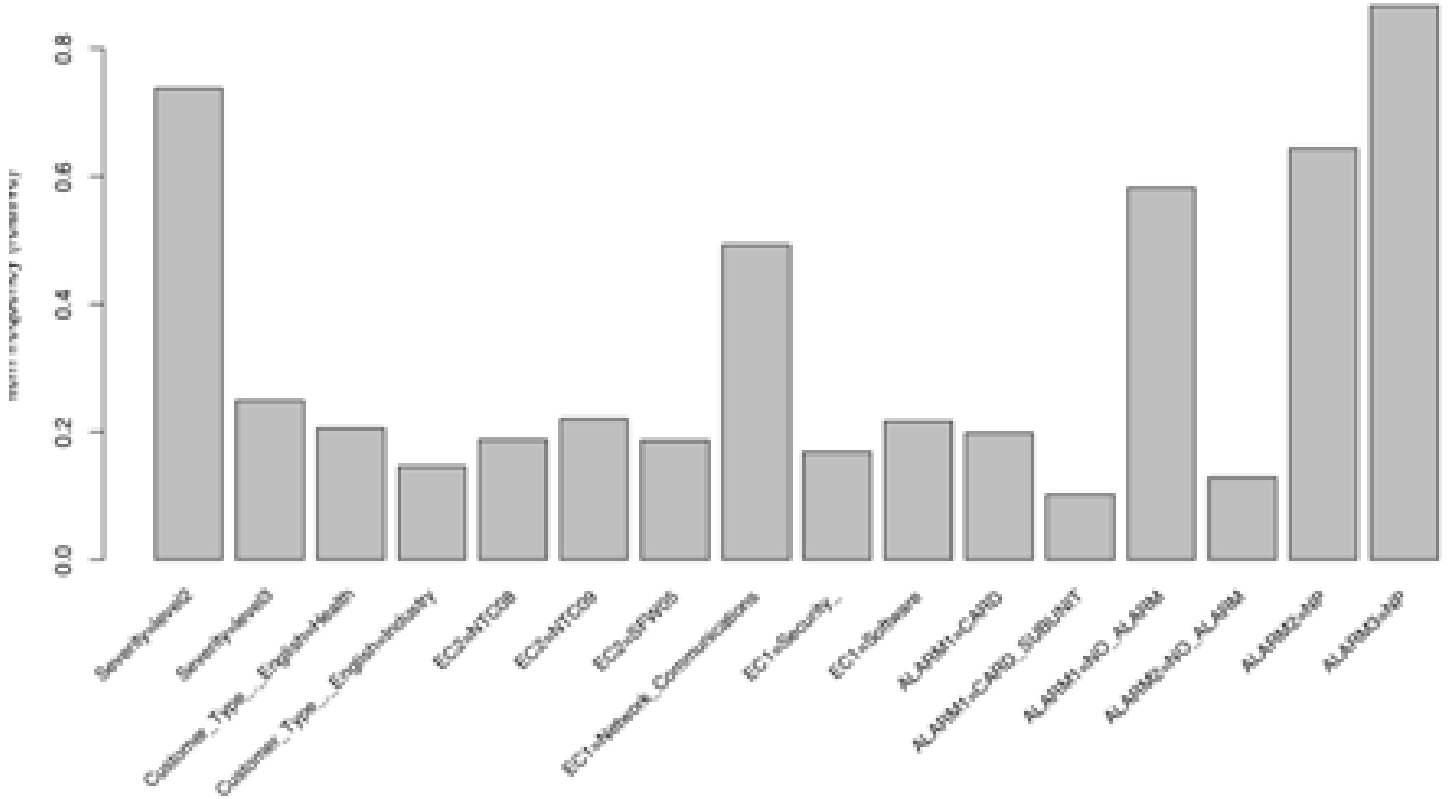



Kenett, R.S. and Salini, S. (2008), "Relative Linkage Disequilibrium Applications to Aircraft Accidents and Operational Risks". *Trans. on Machine Learning and Data Mining*, Vol.1, No 2, pp. 83-96.

# The Telecom Systems Example

| SITE NAME | CCS VERS | SYS. TYPE | GC DUP | GC VER | LAST BOOT & CAUSE | DC_ SNAP | SNAP SHOT | NET | ALARMS | POOL FAULT | RESOURCES | SYSTEM & TASK RESTART | TESTI DATE-TI |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 90006 | 14.66.35 | HDC | | 8.4 | 2-SEP -2007 09:20 | | 0 | | | pl4_tab-177<br>cpn_tab-10 | DTMF-15 | RESET_POWER_UP-1<br>TOTAL_RESTARTS-1 | 11-Sep-2<br>04:00:51 |
| 90009 | 11.11.17 | SX | | 7.23 | AUG -07-20 08:06 AM | | 116 | | | pl4_tab-255<br>pl6_ma-4<br>call_tab-24<br>call_rec-25 | | NMI_WD-1<br>SUSPECT_ACF-1<br>RESET_POWER_UP-1<br>TOTAL_RESTARTS-1 | 11-Sep-20<br>04:03:19 |
| 90021 | 11.11.17 | MEX | | 38.13 | 6-JUN -2006 11:38 | | 0 | | | pl4_tab-235 | | | 11-Sep-<br>04:10:2 |
| 90033 | 11.11.16 | SX | | 7.19 | 14-FEB -2007 05:56 | | 0 | | PCM TIME SLOT | pl4_tab-39 | | | 11-Sep-2<br>04:13:13 |
| 90049 | 11.11.17 | SX | | 7.23 | 4-AUG -2007 13:29 | | 0 | | | status-255<br>features-255<br>timers-255<br>ts_pool-255 | | | 11-Sep-2<br>04:17:20 |
| 90067 | 11.11.17 | SX | | 7.23 | 25-MAY -2007 10:31 | | 0 | | | pl4_def-1<br>pl4_tab-177<br>call_tab-1<br>call_rec-1 | DTMF-9 | OVERLOAD-1 | 11-Sep-<br>04:32: |
| 90098 | 11.11.16 | SX | | 38.13 | 25-JUL -2007 11:47 | | 0 | | | pl4_tab-219<br>call_tab-5 | | | 11-Sep-<br>04:40:3 |
| 90100 | 11.11.16 | SX | | 7.5 | 1-APR -2007 23:22 | | 0 | | | pl4_tab-29<br>pl6_ma-2<br>call_tab-12<br>call_rec-32 | | | 11-Sep-<br>04:42:0 |
| 90105 | 15.68.14 | IPX50 | | 8.5 | 14-AUG -2007 15:54 | | 0 | | CARD SUBUNIT | pl4_tab-106 | | | 11-Sep-<br>04:44:5 |
| 90118 | 11.11.11 | SX | | 38.10 | 23-NOV -2006 16:31 | | 0 | | CARD SUBUNIT | pl4_tab-38 | | | 11-Sep-<br>04:51:0 |
| 90125 | 11.11.16 | SX | | 0.0 | 19-AUG -2007 15:10 | | 0 | | | status-255<br>features-255<br>timers-255<br>*:-3 | | POWER_FAIL-1<br>SUSPECT_ACF-1<br>RESET_POWER_UP-1<br>TOTAL_RESTARTS-1 | 11-Sep-<br>04:58:1 |
| 90126 | 14.66.35 | SVC | | 38.10 | SEP -25-20 11:42 AM | | 0 | | | pl4_def-9<br>pl4_tab-227 | | | 11-Sep-2<br>05:01:16 |
| 90128 | 11.11.16 | SX | | 38.10 | 4-JUN -2006 07:48 | | 0 | | | pl4_tab-19 | | RDY_TMOUT-2<br>OVERLOAD-7<br>POWER_FAIL-12 | 11-Sep-<br>05:04: |

# The Telecom Systems Example



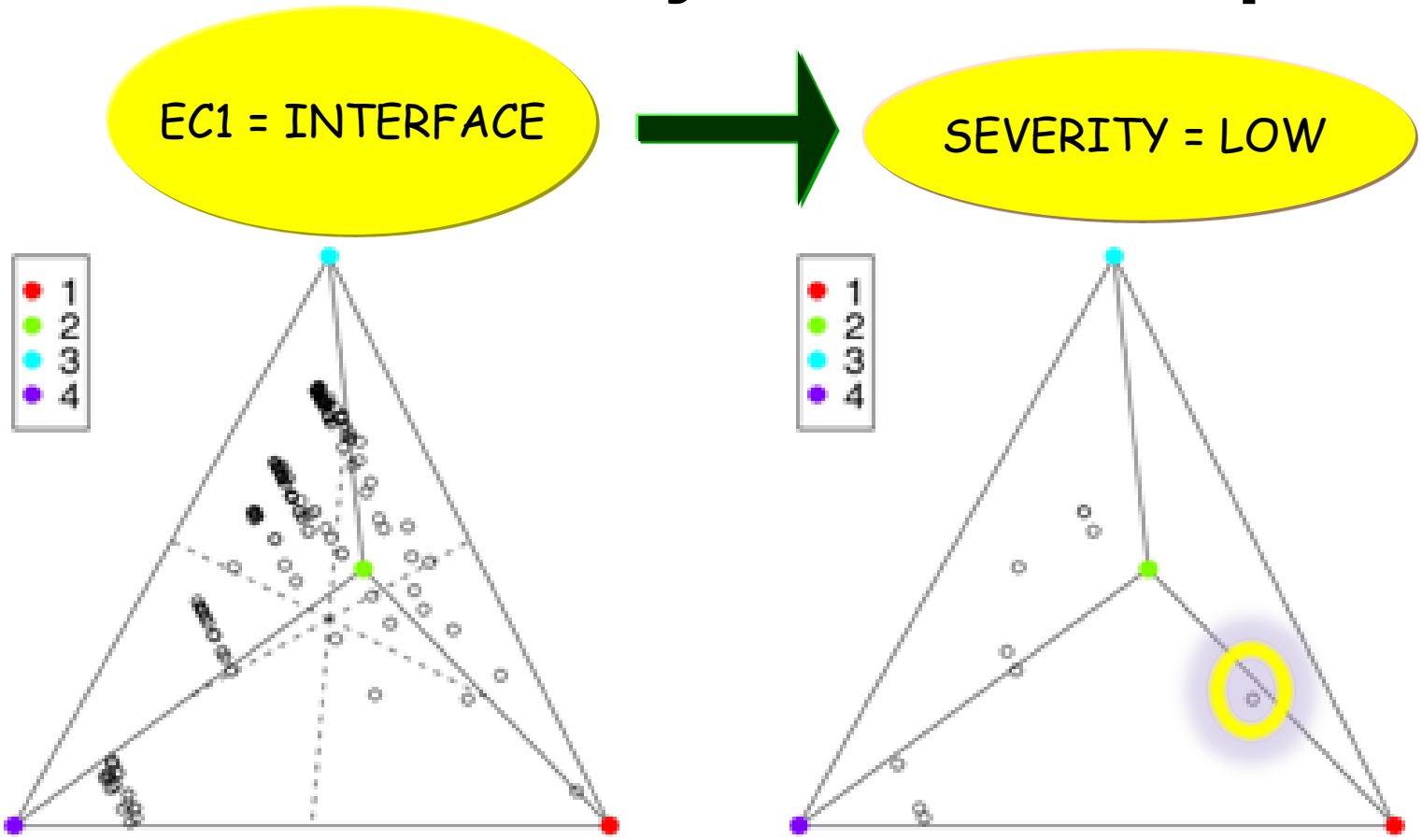

Item Frequency Plot (Support>0.1)

# The Telecom Systems Example

| | lhs | | rhs | support | confidence | lift |
|---|---|---|---|---|---|---|
| 1 | {EC1=Software} | => | {EC2=SFW05} | 0.1864452 | 0.8592593 | 4.608642 |
| 2 | {ALARM2=NO_ALARM} | => | {ALARM1=NO_ALARM} | 0.1285829 | 1.0000000 | 1.717111 |
| 3 | {EC2=SFW05} | => | {EC1=Software} | 0.1864452 | 1.0000000 | 4.608642 |
| 4 | {EC2=SFW05} | => | {Severity=level2} | 0.1864452 | 1.0000000 | 1.354991 |
| 5 | {EC2=NTC08} | => | {EC1=Network_Communications} | 0.1877846 | 1.0000000 | 2.027702 |
| 6 | {EC2=NTC08} | => | {Severity=level2} | 0.1877846 | 1.0000000 | 1.354991 |
| 7 | {EC2=NTC09} | => | {EC1=Network_Communications} | 0.2207340 | 1.0000000 | 2.027702 |
| 8 | {EC2=NTC09} | => | {Severity=level2} | 0.2207340 | 1.0000000 | 1.354991 |
| 9 | {ALARM2=NP} | => | {ALARM3=NP} | 0.6439861 | 1.0000000 | 1.154298 |
| 10 | {EC2=SFW05, ALARM1=NO_ALARM} | => | {EC1=Software} | 0.1090276 | 1.0000000 | 4.608642 |

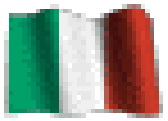
Top 10 rules sorted by RLD of telecom data set

# The Telecom Systems Example




3D Simplex Representation for 200 rules of telecom data set and for the top 10 rules sorted by RLD

# The Italian Case Study



- Lot_id :  2 lots of the same product
- Sequence : for each lot, 25 wafers are tested (sequence 1-25/day)
- Date: measures are available over 2 days
- Site: where measures are collected (no info about the xy coordinate)
- PARAM_1 – 56 : 56 electrical parameters that are jointly measured
- Equip: the gauge used for measurement

# The Italian Case Study

| Sequence | Site | PARAM_1 | PARAM_2 | PARAM_3 | PARAM_4 | PARAM_5 | PARAM_6 | PARAM_7 | PARAM_8 | PARAM_9 | [illegible]_55 | PARAM_56 | Product | Date | Limfile | Equip |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 15 | 13.075 | 0.67852 | 1.7088 | 5.7397 | 11.965 | -0.86602 | -0.82173 | -4.316 | -10.653 | [illegible]24 | 12.196 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 1 | 63 | 11.228 | 0.67441 | 1.7818 | 6.1232 | 11.857 | -0.86445 | -0.82283 | -3.1163 | -10.607 | [illegible]1 | 11.214 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 1 | 69 | 12.932 | 0.67188 | 1.7327 | 3.8422 | 11.608 | -0.87266 | -0.83701 | -2.9699 | -10.494 | [illegible]08 | 11.667 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 1 | 55 | 11.453 | 0.68008 | 1.7626 | 4.328 | 11.906 | -0.86484 | -0.80937 | -4.1754 | -10.591 | [illegible].065 | 11.556 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 1 | 100 | 12.559 | 0.68203 | 1.7107 | 8.7719 | 11.824 | -0.87344 | -0.80663 | -4.4965 | -10.564 | [illegible]954 | 58.185 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 2 | 15 | 12.663 | 0.67969 | 1.7393 | 4.533 | 11.658 | -0.875 | -0.82876 | -3.2058 | -10.615 | [illegible]84 | 11.779 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 2 | 63 | 11.209 | 0.67227 | 1.7951 | 2.6564 | 11.809 | -0.86953 | -0.81744 | -0.88106 | -10.578 | [illegible]6 | 10.839 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 2 | 69 | 12.704 | 0.67383 | 1.7716 | 2.4721 | 11.957 | -0.875 | -0.84255 | -2.0976 | -10.529 | [illegible]104 | 11.491 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 2 | 55 | 11.221 | 0.67578 | 1.778 | 2.7333 | 11.875 | -0.87578 | -0.80597 | -1.6098 | -10.583 | [illegible]989 | 11.417 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 2 | 100 | 11.934 | 0.67773 | 1.7677 | 3.3927 | 11.861 | -0.87969 | -0.81858 | -2.2595 | -10.599 | [illegible]72 | 11.447 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 3 | 15 | 12.878 | 0.67656 | 1.7264 | 3.2056 | 11.792 | -0.87773 | -0.81234 | -2.4831 | -10.617 | [illegible]9 | 11.866 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 3 | 63 | 11.237 | 0.67461 | 1.7853 | 1.0371 | 11.843 | -0.87773 | -0.80511 | -0.69568 | -10.609 | [illegible]213 | 10.942 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 3 | 69 | 12.75 | 0.67383 | 1.7345 | 2.1477 | 11.815 | -0.88359 | -0.83297 | -1.8404 | -10.488 | [illegible].336 | 11.338 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 3 | 55 | 11.462 | 0.67773 | 1.7555 | 1.5873 | 11.832 | -0.88047 | -0.79748 | -1.1981 | -10.577 | [illegible]68 | 11.111 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 3 | 100 | 12.293 | 0.68008 | 1.7368 | 2.6361 | 11.837 | -0.88359 | -0.79557 | -2.0674 | -10.56 | [illegible]2 | 11.589 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 4 | 15 | 12.311 | 0.68203 | 1.7251 | 5.8527 | 11.742 | -0.87813 | -0.8151 | -1.2752 | -10.635 | [illegible]994 | 11.132 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 4 | 63 | 11 | 0.67656 | 1.7724 | 1.345 | 11.845 | -0.87461 | -0.80448 | -0.49988 | -10.632 | [illegible].531 | 11.338 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 4 | 69 | 12.522 | 0.66914 | 1.7643 | 3.9448 | 11.892 | -0.87773 | -0.8331 | -1.4123 | -10.517 | [illegible]?51 | 11.591 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 4 | 55 | 11.105 | 0.67813 | 1.7618 | 1.0714 | 11.745 | -0.87891 | -0.80548 | -0.82438 | -10.622 | [illegible]?8 | 11.091 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 4 | 100 | 12.226 | 0.67773 | 1.7455 | 1.4786 | 11.843 | -0.88203 | -0.81382 | -1.2918 | -10.583 | [illegible].08 | 10.949 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 5 | 15 | 12.573 | 0.67617 | 1.728 | 1.2996 | 11.865 | -0.86914 | -0.82227 | -0.92897 | -10.64 | [illegible].069 | 11.88 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 5 | 63 | 11.355 | 0.67461 | 1.7763 | 1.456 | 11.796 | -0.87383 | -0.8075 | -0.62439 | -10.53 | [illegible]?83 | 11.864 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 5 | 69 | 12.728 | 0.67188 | 1.7649 | 2.0677 | 11.796 | -0.87852 | -0.83684 | -2.0241 | -10.531 | [illegible]97 | 11.132 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 5 | 55 | 11.368 | 0.67422 | 1.7729 | 1.7251 | 11.766 | -0.87578 | -0.79652 | -0.65173 | -10.594 | [illegible].5 | 11.856 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 5 | 100 | 11.95 | 0.67969 | 1.7411 | 2.6781 | 11.764 | -0.87969 | -0.80627 | -1.1112 | -10.622 | [illegible]319 | 11.33 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 6 | 15 | 12.68 | 0.68203 | 1.732 | 1.5987 | 11.795 | -0.87852 | -0.82131 | -0.76203 | -10.64 | [illegible]376 | 11.69 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 6 | 63 | 11.185 | 0.67695 | 1.7758 | 1.8005 | 11.8 | -0.87695 | -0.80476 | -0.61824 | -10.639 | [illegible]65 | 11.377 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 6 | 69 | 12.804 | 0.67266 | 1.7428 | 1.6406 | 11.793 | -0.8793 | -0.83267 | -1.4923 | -10.493 | [illegible]5 | 12.028 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| 6 | 55 | 11.302 | 0.675 | 1.7791 | 1.2361 | 11.782 | -0.87891 | -0.80488 | -0.72644 | -10.598 | [illegible]47 | 11.325 | FW6FCAN | 20100831 | CMOST11I | T8412-s68 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] | 0.6707 | [illegible] | 3.0371 | [illegible] | 0.87266 | -0.81[illegible] | [illegible]226 | -10.6[illegible] | [illegible] | 11.67 | [illegible] | 20100831 | C[illegible] | [illegible]12-s68 |

# Data characteristic

## Distributions

### Lot_id

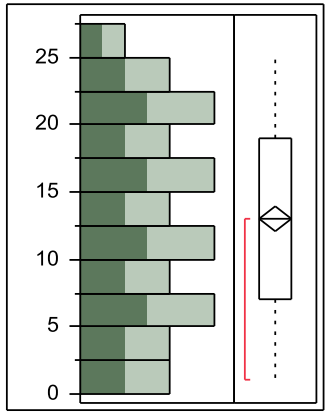

**Frequencies**

| Level | Count | Prob |
|---|---|---|
| A026250 | 125 | 0.50000 |
| A026286 | 125 | 0.50000 |
| Total | 250 | 1.00000 |

N Missing 0

2 Levels

### Sequence



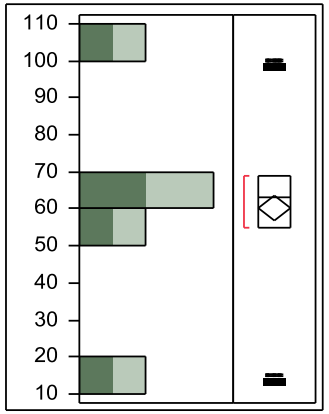

**Quantiles**

| | | |
|---|---|---|
| 100.0% | maximum | 25.000 |
| 99.5% | | 25.000 |
| 97.5% | | 25.000 |
| 90.0% | | 23.000 |
| 75.0% | quartile | 19.000 |
| 50.0% | median | 13.000 |
| 25.0% | quartile | 7.000 |
| 10.0% | | 3.000 |
| 2.5% | | 1.000 |
| 0.5% | | 1.000 |
| 0.0% | minimum | 1.000 |

**Moments**

| | |
|---|---|
| Mean | 13 |
| Std Dev | 7.2255682 |
| Std Err Mean | 0.4569851 |
| Upper 95% Mean | 13.900049 |
| Lower 95% Mean | 12.099951 |
| N | 250 |

### Site



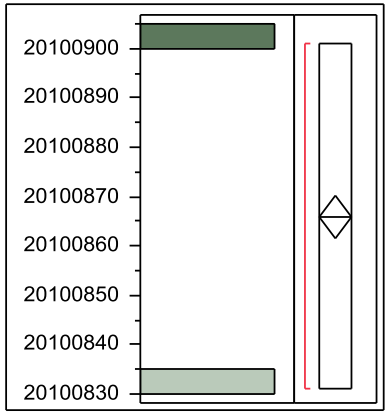

**Quantiles**

| | | |
|---|---|---|
| 100.0% | maximum | 100.00 |
| 99.5% | | 100.00 |
| 97.5% | | 100.00 |
| 90.0% | | 100.00 |
| 75.0% | quartile | 69.00 |
| 50.0% | median | 63.00 |
| 25.0% | quartile | 55.00 |
| 10.0% | | 15.00 |
| 2.5% | | 15.00 |
| 0.5% | | 15.00 |
| 0.0% | minimum | 15.00 |

**Moments**

| | |
|---|---|
| Mean | 60.4 |
| Std Dev | 27.401521 |
| Std Err Mean | 1.7330244 |
| Upper 95% Mean | 63.813255 |
| Lower 95% Mean | 56.986745 |
| N | 250 |

### Date



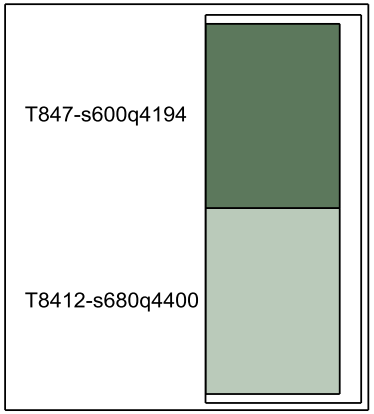

**Quantiles**

| | | |
|---|---|---|
| 100.0% | maximum | 20100901 |
| 99.5% | | 20100901 |
| 97.5% | | 20100901 |
| 90.0% | | 20100901 |
| 75.0% | quartile | 20100901 |
| 50.0% | median | 20100866 |
| 25.0% | quartile | 20100831 |
| 10.0% | | 20100831 |
| 2.5% | | 20100831 |
| 0.5% | | 20100831 |
| 0.0% | minimum | 20100831 |

**Moments**

| | |
|---|---|
| Mean | 20100866 |
| Std Dev | 35.070211 |
| Std Err Mean | 2.2180349 |
| Upper 95% Mean | 20100870 |
| Lower 95% Mean | 20100862 |
| N | 250 |

### Equip




**Frequencies**

| Level | Count | Prob |
|---|---|---|
| T8412-s680q4400 | 125 | 0.50000 |
| T847-s600q4194 | 125 | 0.50000 |
| Total | 250 | 1.00000 |

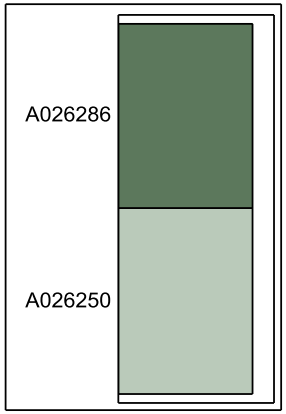
N Missing 0

2 Levels

# Data characteristic

## Distributions

### Lot_id



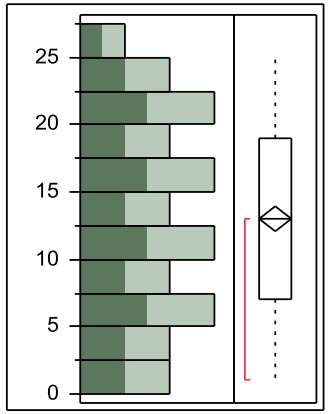

**Frequencies**

| Level | Count | Prob |
|---|---|---|
| A026250 | 125 | 0.50000 |
| A026286 | 125 | 0.50000 |
| Total | 250 | 1.00000 |

N Missing 0

2 Levels

### Sequence



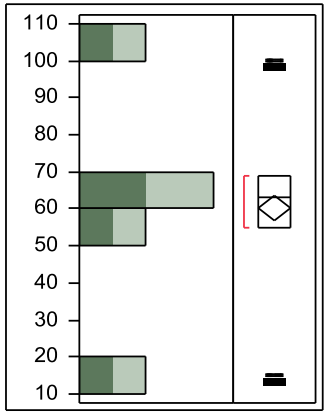

**Quantiles**

| | | |
|---|---|---|
| 100.0% | maximum | 25.000 |
| 99.5% | | 25.000 |
| 97.5% | | 25.000 |
| 90.0% | | 23.000 |
| 75.0% | quartile | 19.000 |
| 50.0% | median | 13.000 |
| 25.0% | quartile | 7.000 |
| 10.0% | | 3.000 |
| 2.5% | | 1.000 |
| 0.5% | | 1.000 |
| 0.0% | minimum | 1.000 |

**Moments**

| | |
|---|---|
| Mean | 13 |
| Std Dev | 7.2255682 |
| Std Err Mean | 0.4569851 |
| Upper 95% Mean | 13.900049 |
| Lower 95% Mean | 12.099951 |
| N | 250 |

### Site



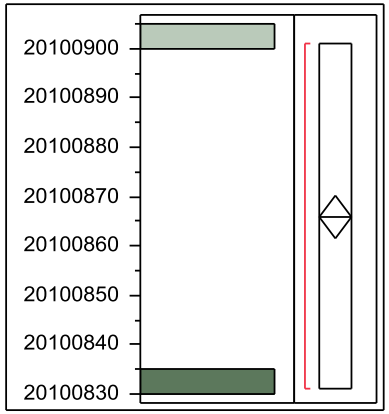

**Quantiles**

| | | |
|---|---|---|
| 100.0% | maximum | 100.00 |
| 99.5% | | 100.00 |
| 97.5% | | 100.00 |
| 90.0% | | 100.00 |
| 75.0% | quartile | 69.00 |
| 50.0% | median | 63.00 |
| 25.0% | quartile | 55.00 |
| 10.0% | | 15.00 |
| 2.5% | | 15.00 |
| 0.5% | | 15.00 |
| 0.0% | minimum | 15.00 |

**Moments**

| | |
|---|---|
| Mean | 60.4 |
| Std Dev | 27.401521 |
| Std Err Mean | 1.7330244 |
| Upper 95% Mean | 63.813255 |
| Lower 95% Mean | 56.986745 |
| N | 250 |

### Date




**Quantiles**

| | | |
|---|---|---|
| 100.0% | maximum | 20100901 |
| 99.5% | | 20100901 |
| 97.5% | | 20100901 |
| 90.0% | | 20100901 |
| 75.0% | quartile | 20100901 |
| 50.0% | median | 20100866 |
| 25.0% | quartile | 20100831 |
| 10.0% | | 20100831 |
| 2.5% | | 20100831 |
| 0.5% | | 20100831 |
| 0.0% | minimum | 20100831 |

**Moments**

| | |
|---|---|
| Mean | 20100866 |
| Std Dev | 35.070211 |
| Std Err Mean | 2.2180349 |
| Upper 95% Mean | 20100870 |
| Lower 95% Mean | 20100862 |
| N | 250 |

### Equip

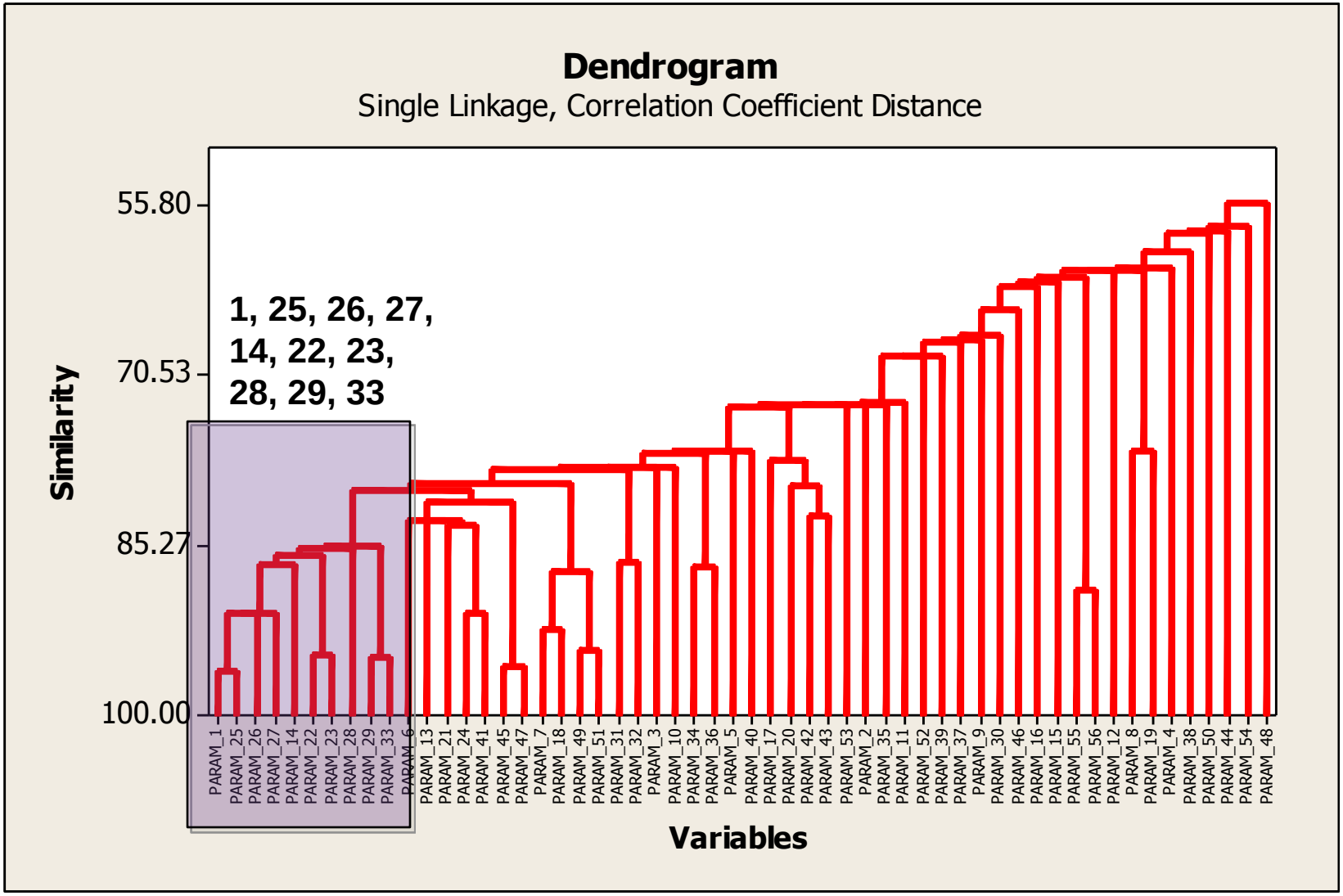

**Frequencies**

| Level | Count | Prob |
|---|---|---|
| T8412-s680q4400 | 125 | 0.50000 |
| T847-s600q4194 | 125 | 0.50000 |
| Total | 250 | 1.00000 |

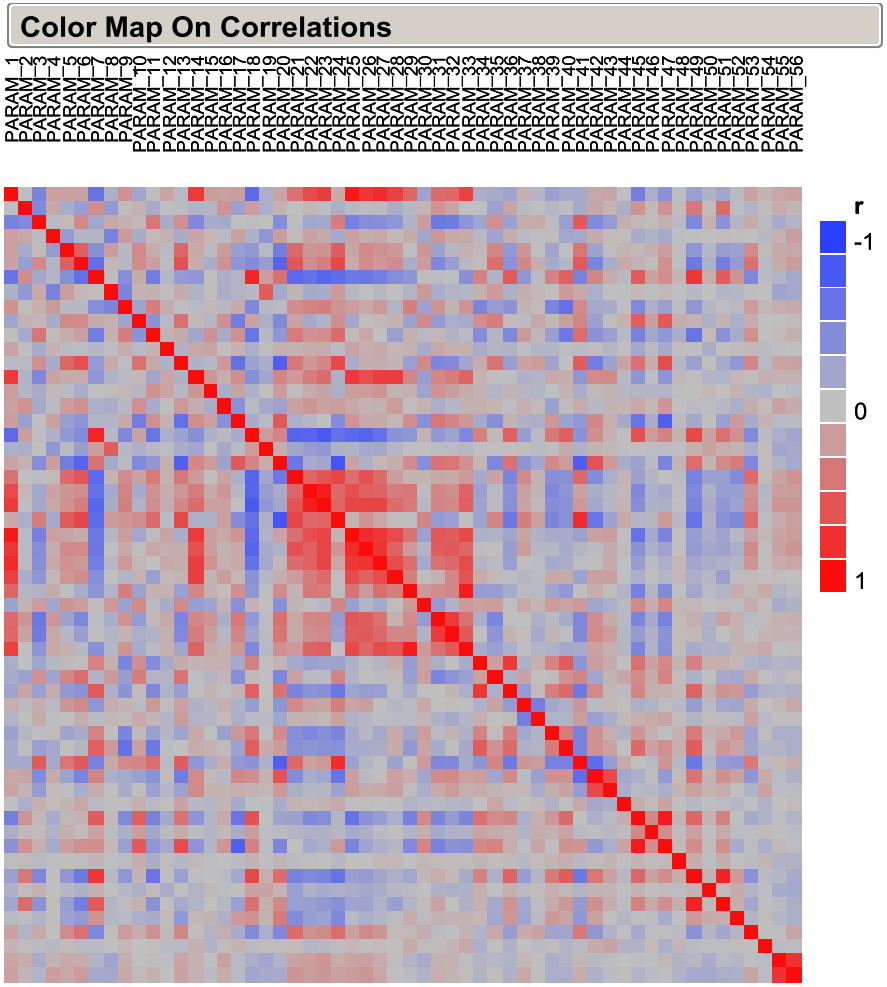
N Missing 0

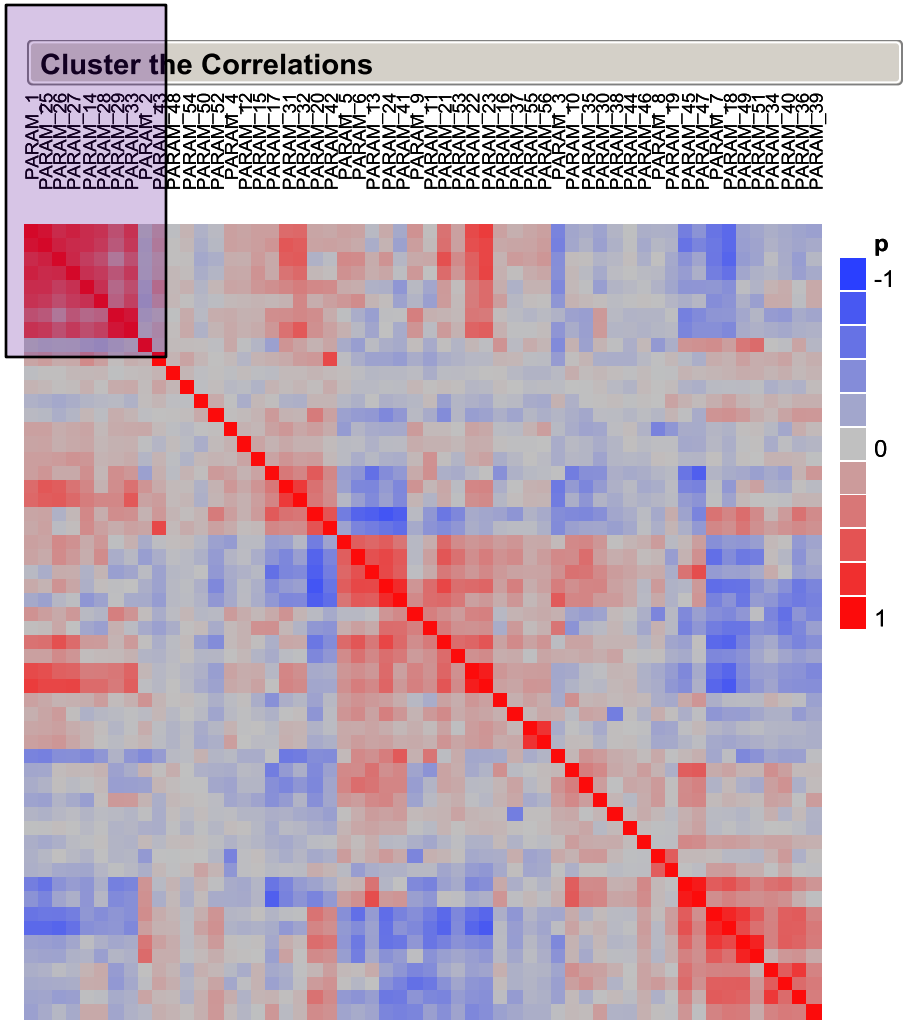
2 Levels

# Cluster Analysis of the 56 Electrical Parameters

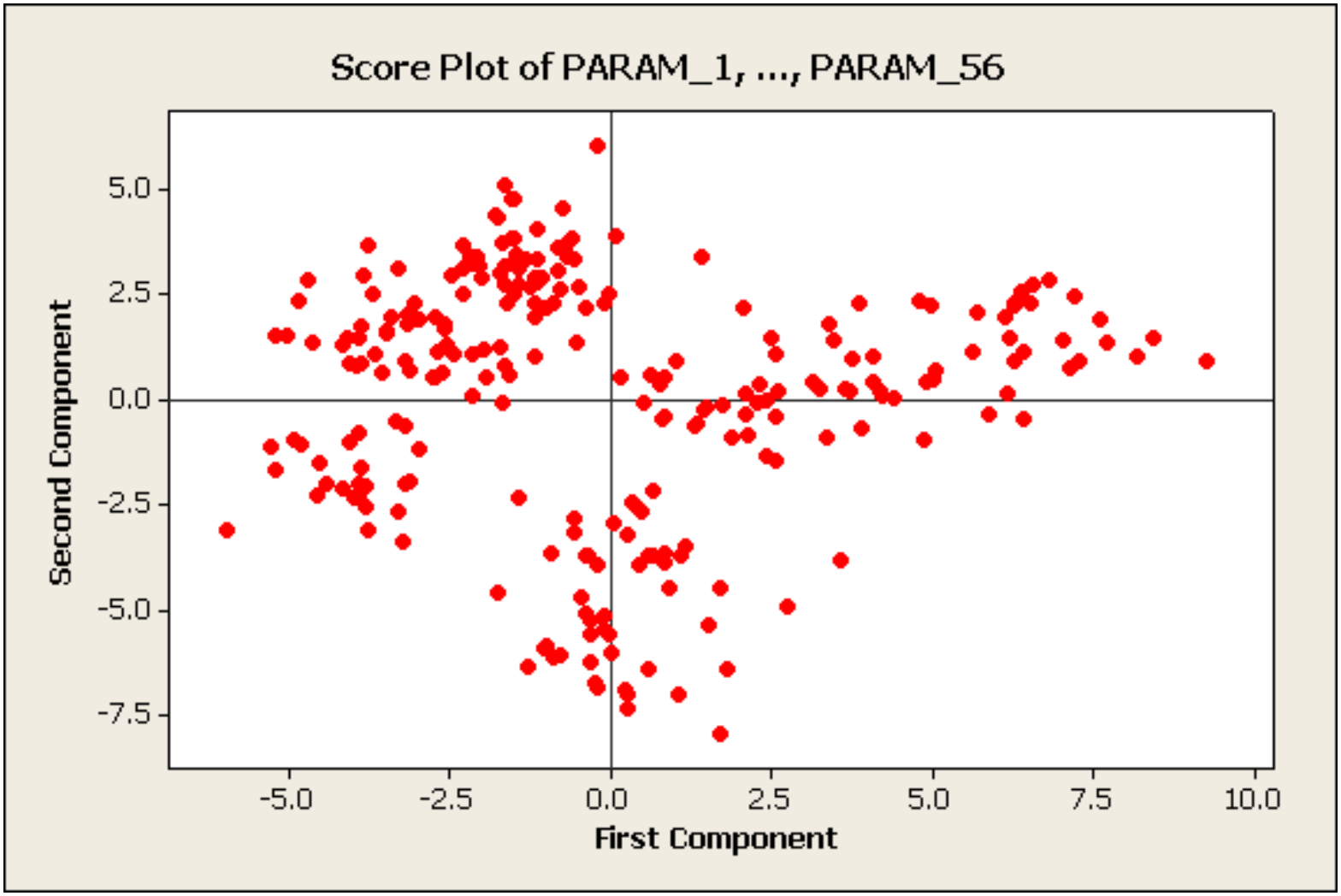


# Cluster Analysis of the 56 Electrical Parameters

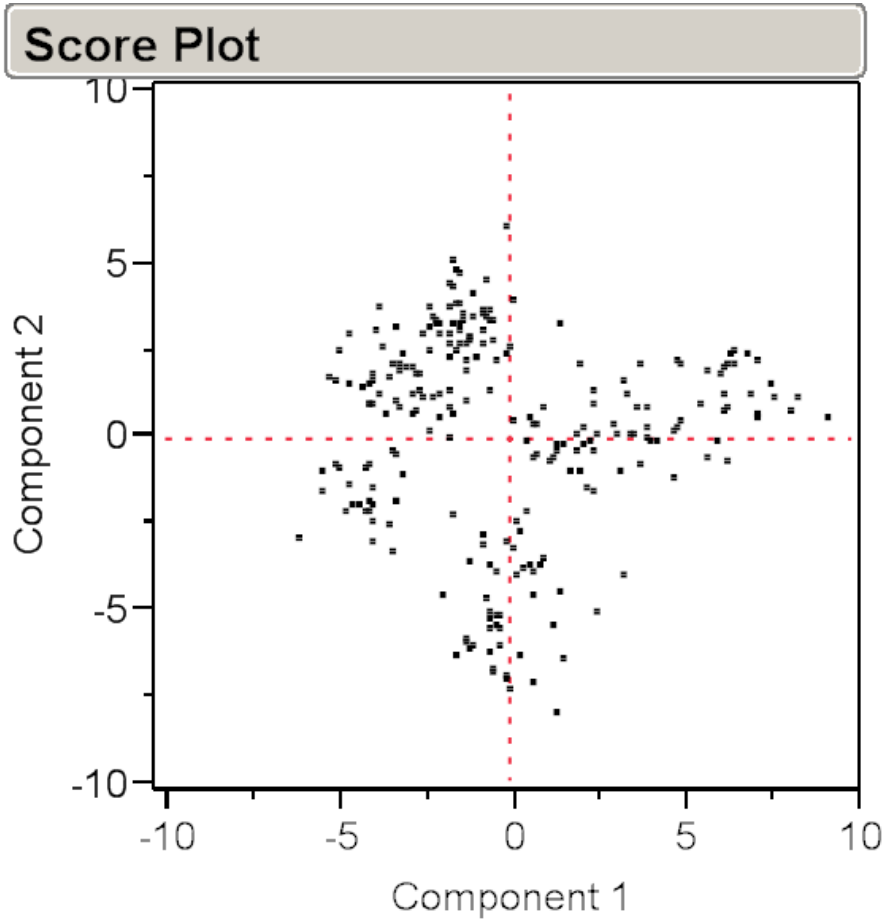


# Principal Components Analysis of the 56 Electrical Parameters

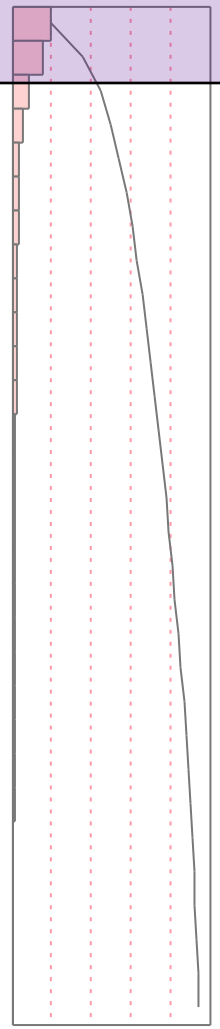


# Principal Components Analysis of the 56 Electrical Parameters

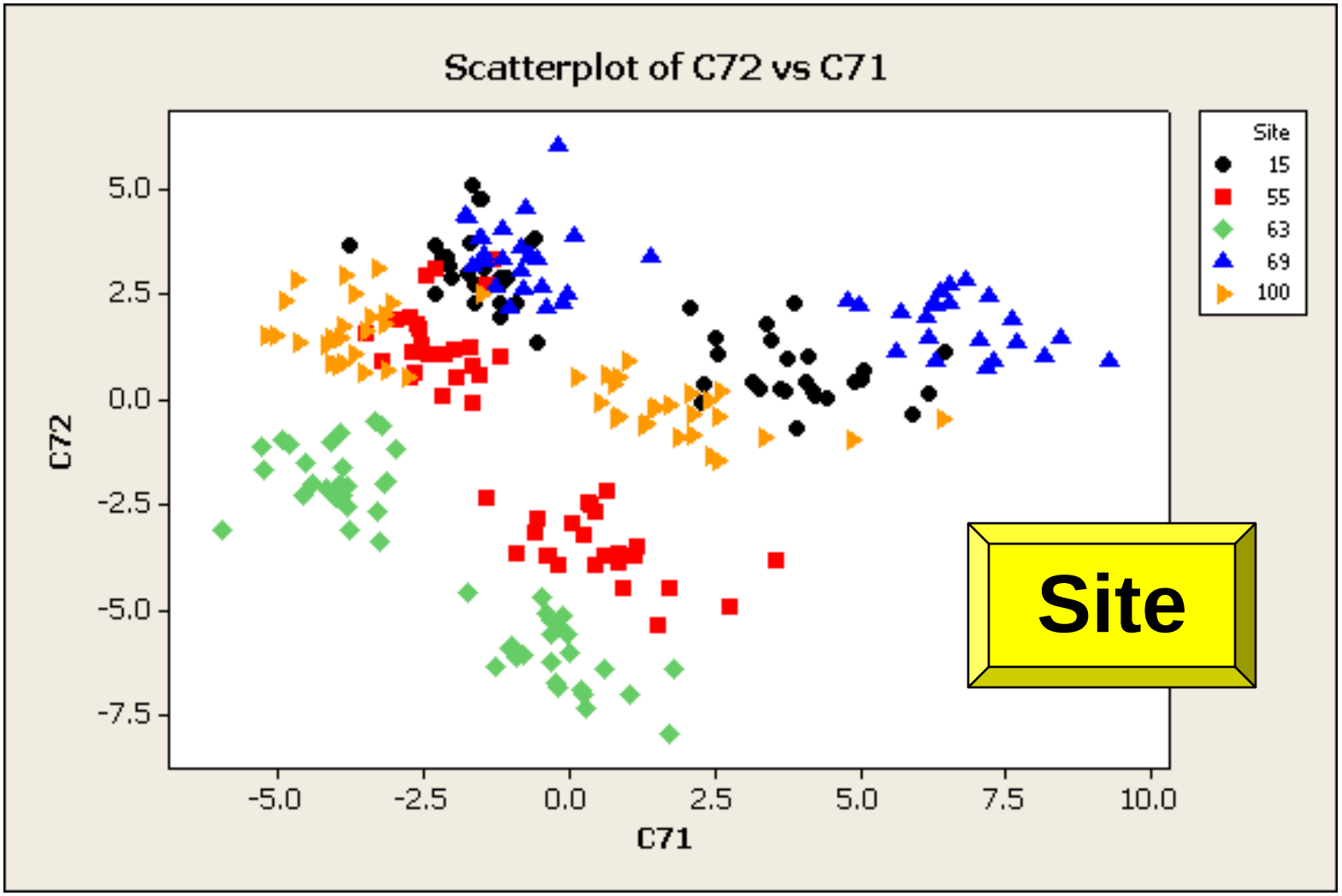



**Principal Components: on Correlations**

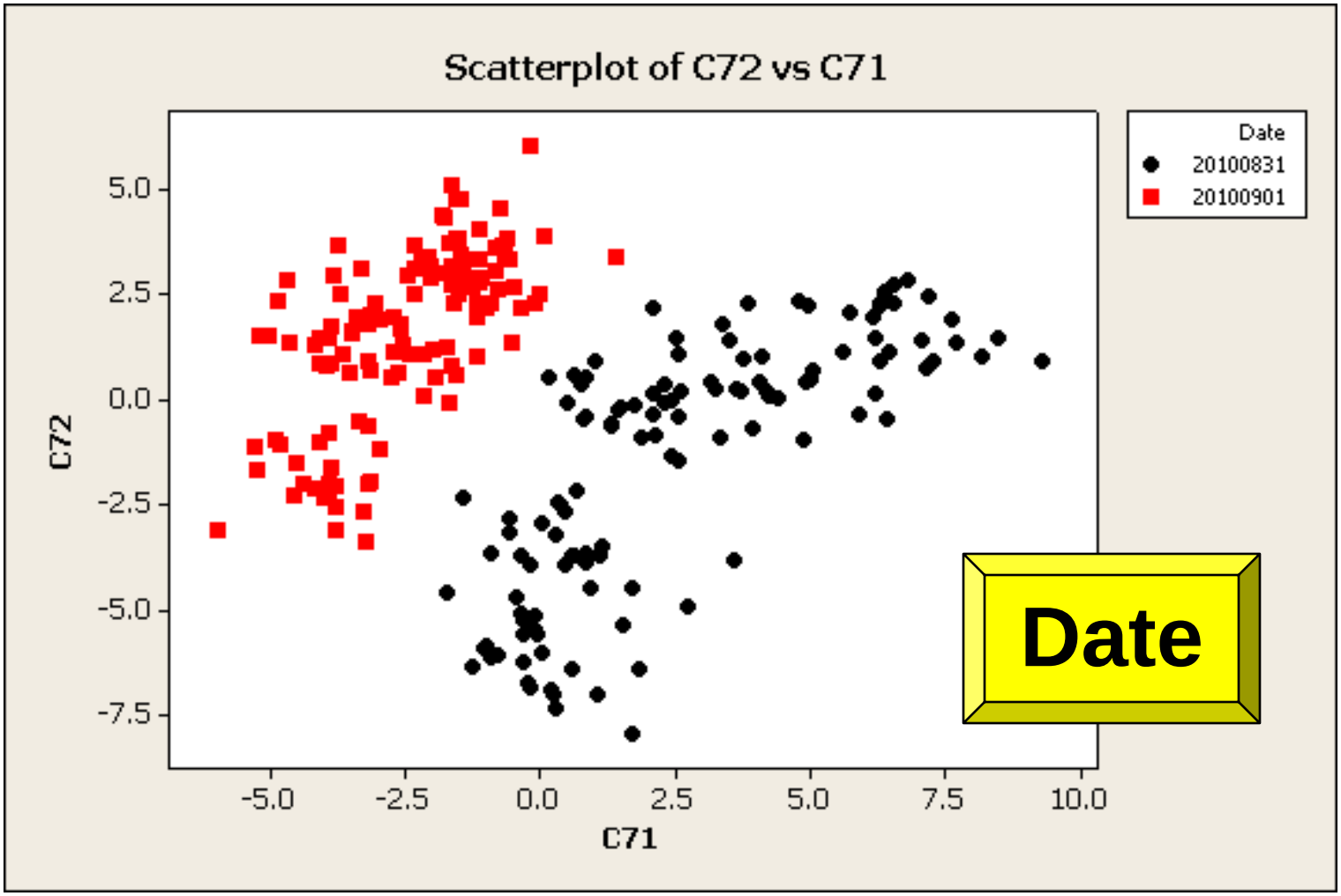
| Number | Eigenvalue | Percent | Cum Percent |
|---|---|---|---|
| 1 | 11.2217 | 20.039 | 20.039 |
| 2 | 8.8963 | 15.886 | 35.925 |
| 3 | 4.8562 | 8.672 | 44.597 |
| 4 | 3.0469 | 5.441 | 50.038 |
| 5 | 2.2017 | 3.932 | 53.969 |
| 6 | 2.0318 | 3.628 | 57.597 |
| 7 | 1.7607 | 3.144 | 60.742 |
| 8 | 1.4627 | 2.612 | 63.354 |
| 9 | 1.4129 | 2.523 | 65.877 |
| 10 | 1.3521 | 2.414 | 68.291 |
| 11 | 1.2776 | 2.281 | 70.572 |
| 12 | 1.1444 | 2.044 | 72.616 |
| 13 | 1.0733 | 1.917 | 74.533 |
| 14 | 0.9921 | 1.772 | 76.304 |
| 15 | 0.9468 | 1.691 | 77.995 |
| 16 | 0.9259 | 1.653 | 79.648 |
| 17 | 0.9145 | 1.633 | 81.282 |
| 18 | 0.8648 | 1.544 | 82.826 |
| 19 | 0.7987 | 1.426 | 84.252 |
| 20 | 0.7571 | 1.352 | 85.604 |
| 21 | 0.7350 | 1.312 | 86.917 |
| 22 | 0.6440 | 1.150 | 88.067 |
| 23 | 0.5943 | 1.061 | 89.128 |
| 24 | 0.5848 | 1.044 | 90.172 |
| 25 | 0.5308 | 0.948 | 91.120 |
| 26 | 0.5037 | 0.900 | 92.020 |
| 27 | 0.4643 | 0.829 | 92.849 |
| 28 | 0.4298 | 0.767 | 93.616 |
| 29 | 0.3672 | 0.656 | 94.272 |
| 30 | 0.3357 | 0.599 | 94.871 |



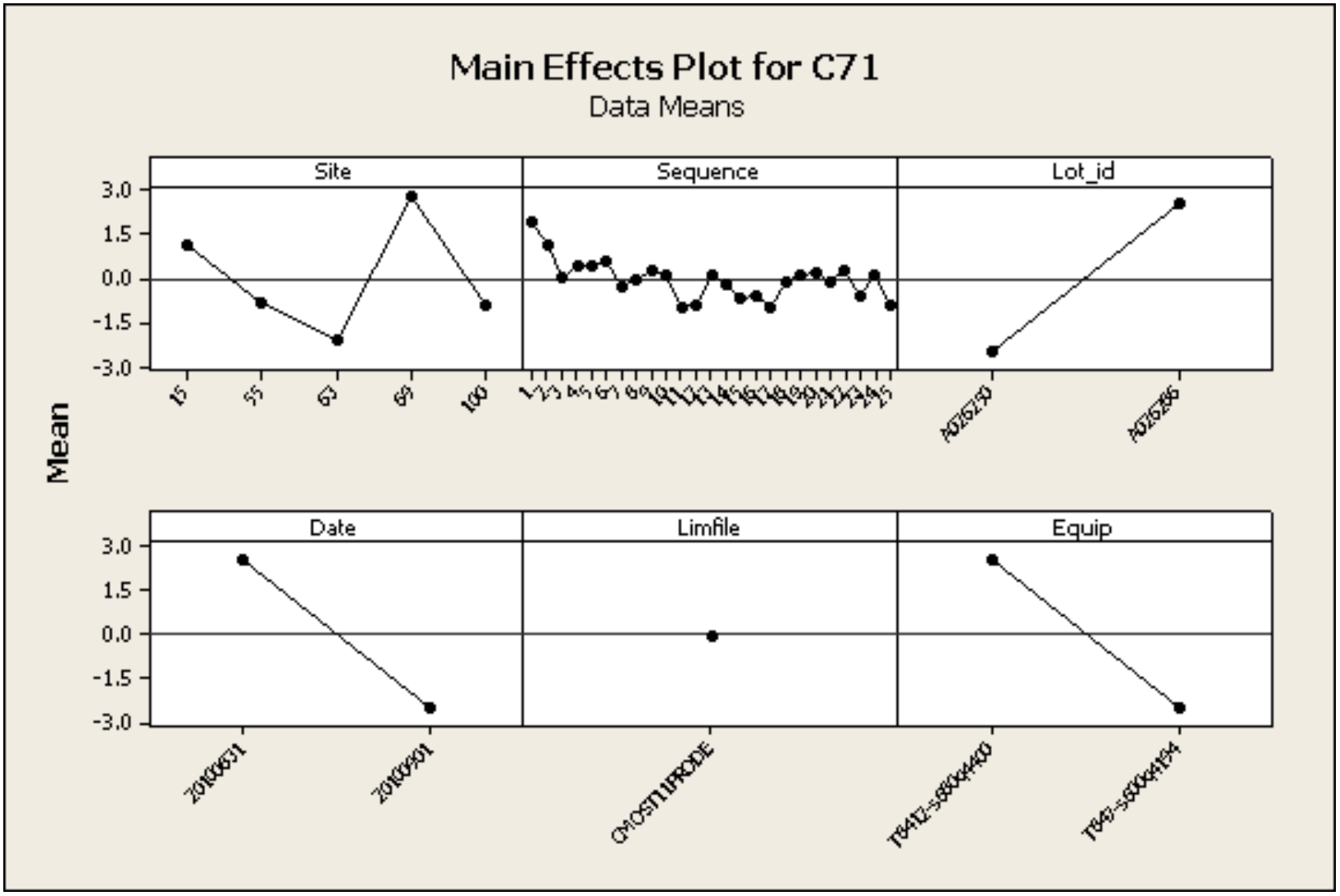
# Principal Components Analysis of the 56 Electrical Parameters



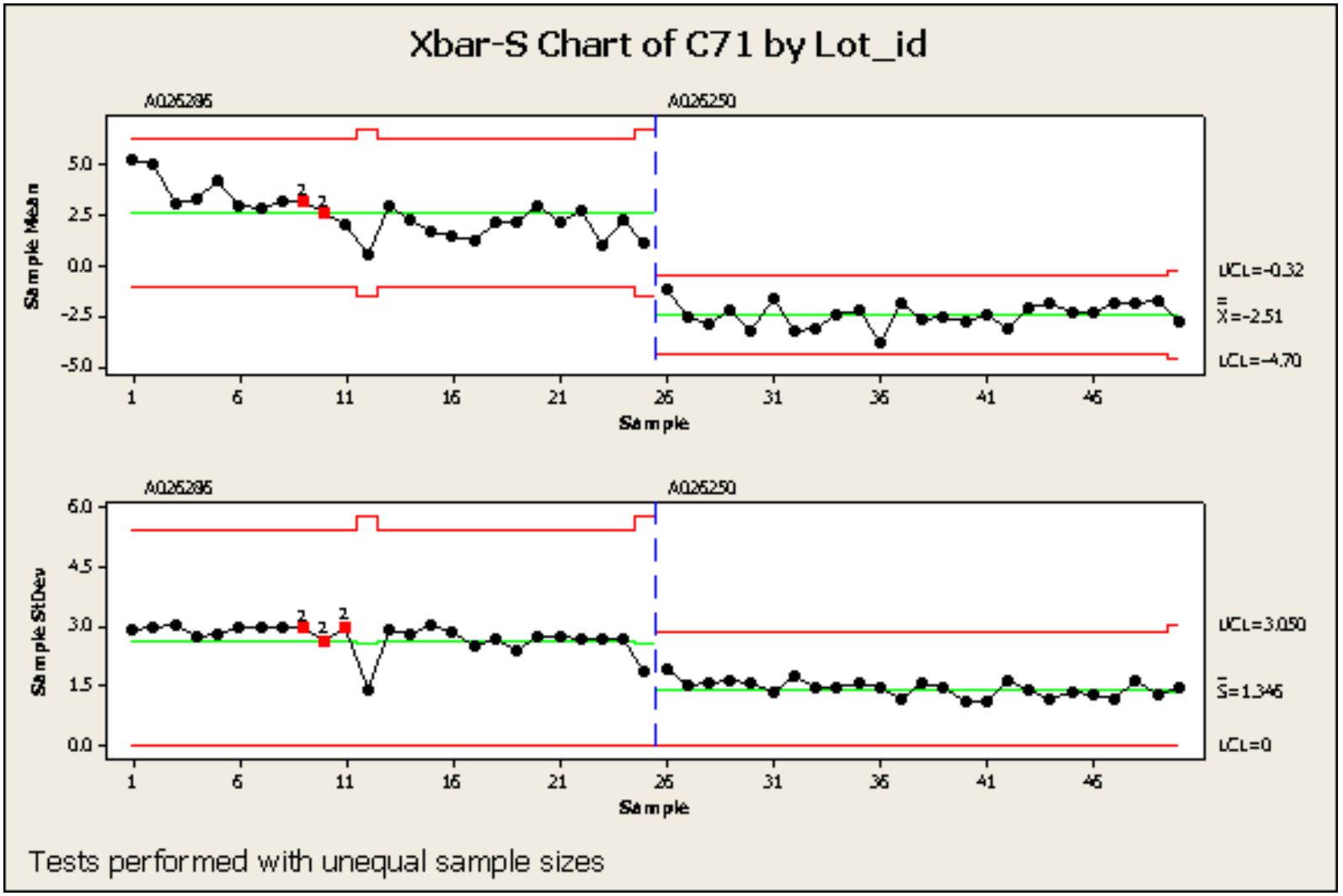
# Principal Components Analysis of the 56 Electrical Parameters



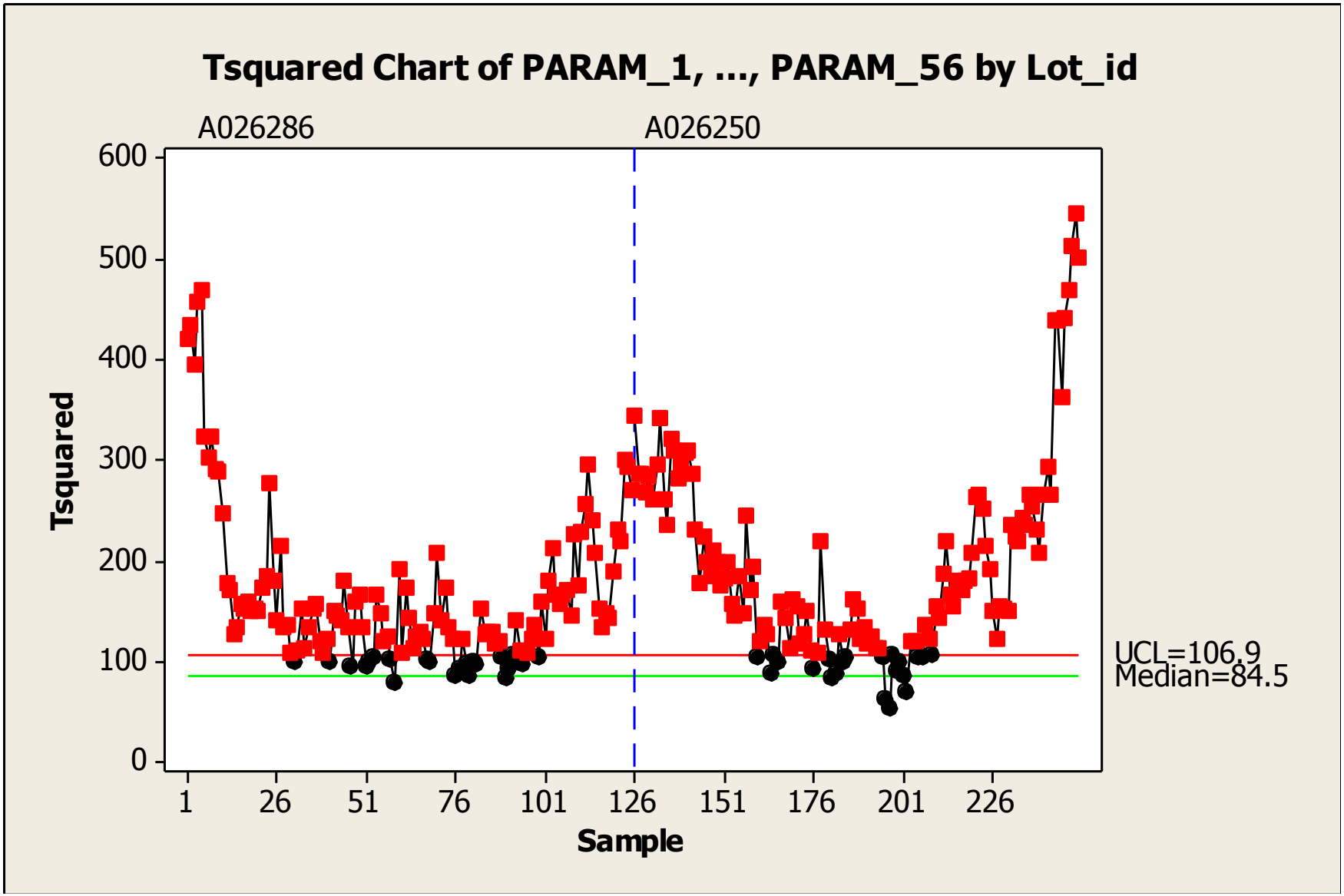
# Main Effect Plots of First Principal Component



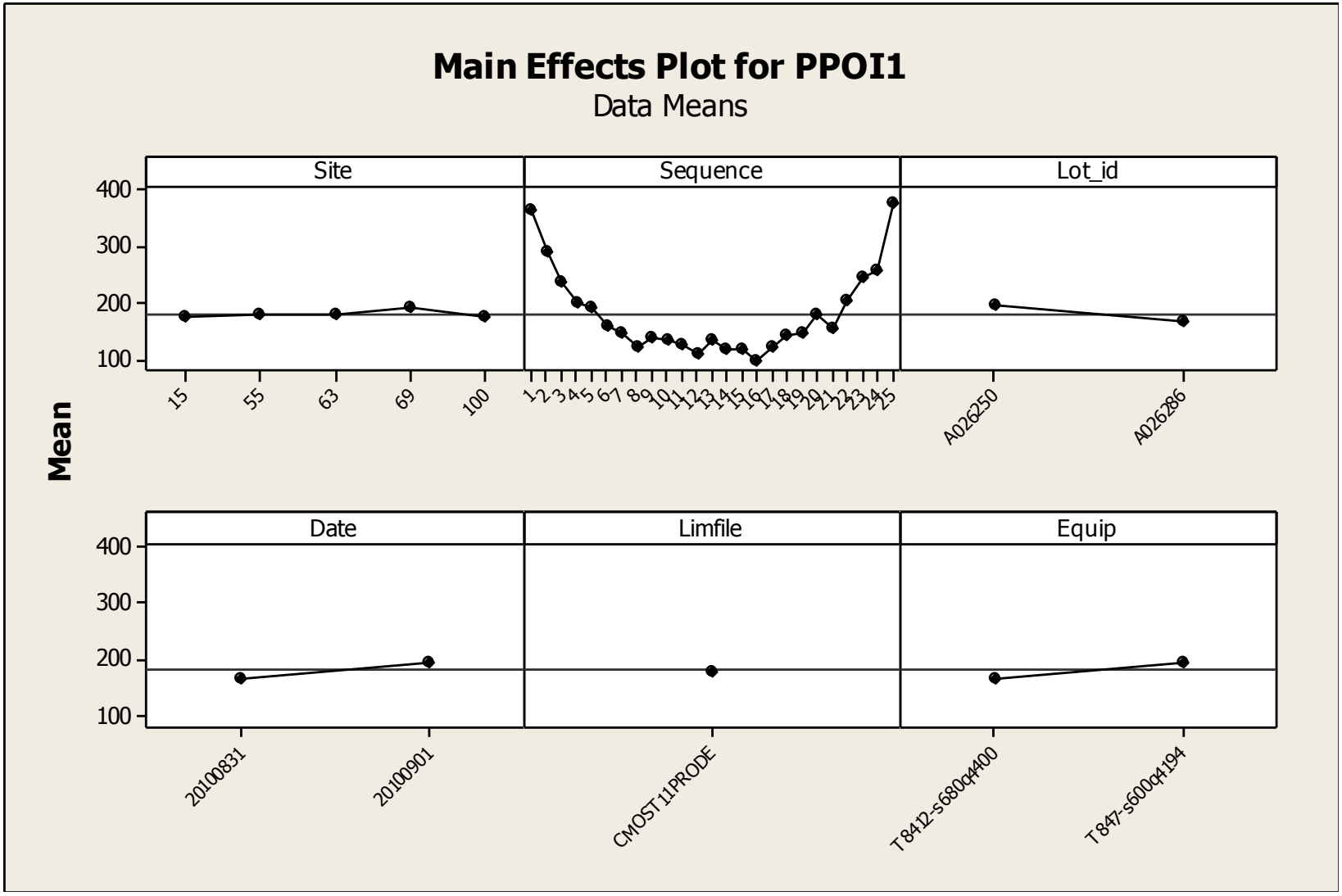
# Xbar-S Control Chart of First Principal Component



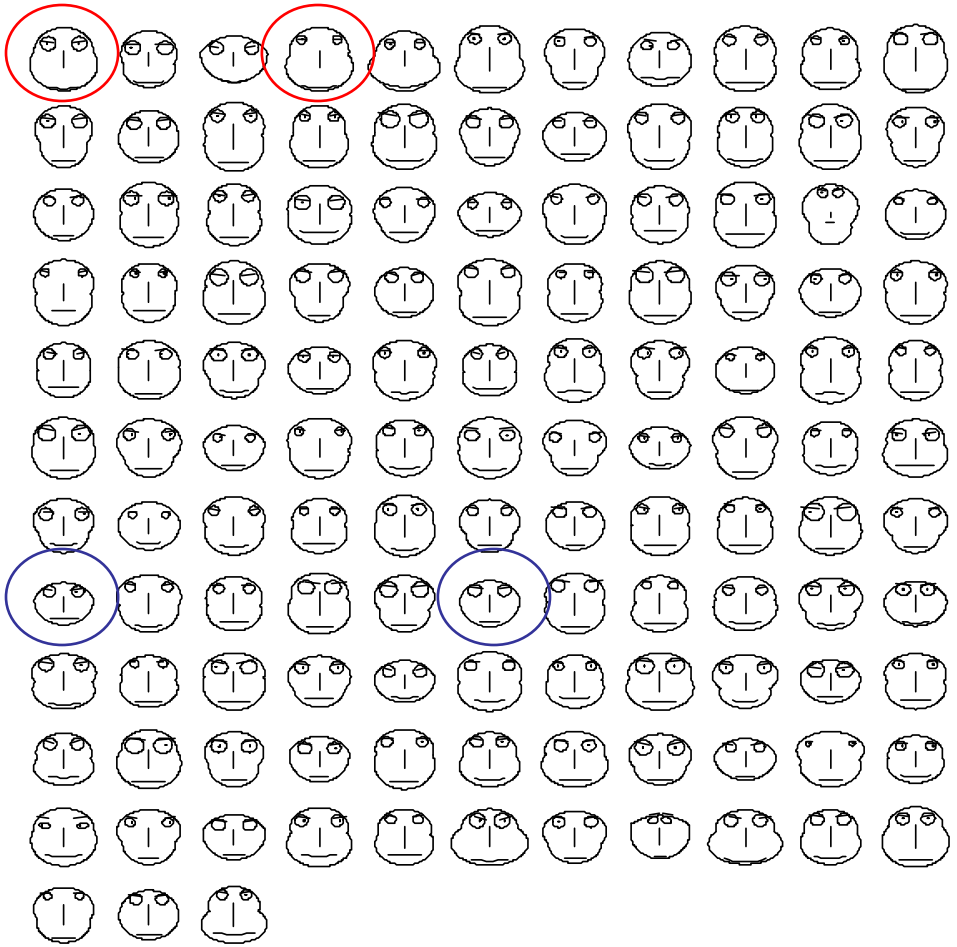
# $T^2$ Control Chart for 56 Parameters



# Main Effect Plots for $T^2$



# Star Plots and Chernoff Faces

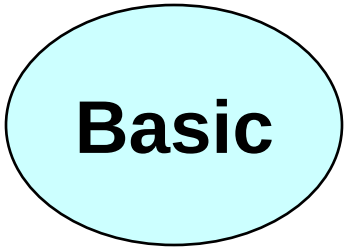



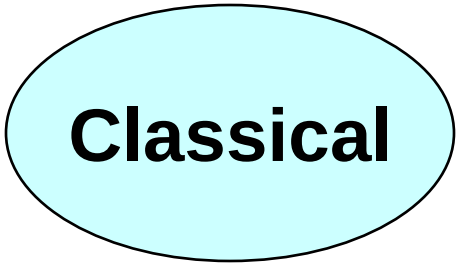
1 Width of center 2 Top vs. Bottom width (height of split) 3 Height of Face 4 Width of top half of face 5 Width of bottom half of face 6 Length of Nose 7 Height of Mouth 8 Curvature of Mouth (abs < 9) 9 Width of Mouth 10 Height of Eyes 11 Distance between Eyes (.5-.9) 12 Angle of Eyes/Eyebrows 13 Circle/Ellipse of Eyes 14 Size of Eyes 15 Position Left/Right of Eyeballs/Eyebrows

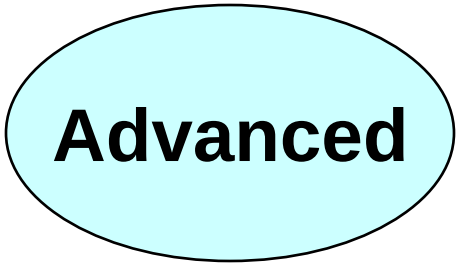
**A026250**

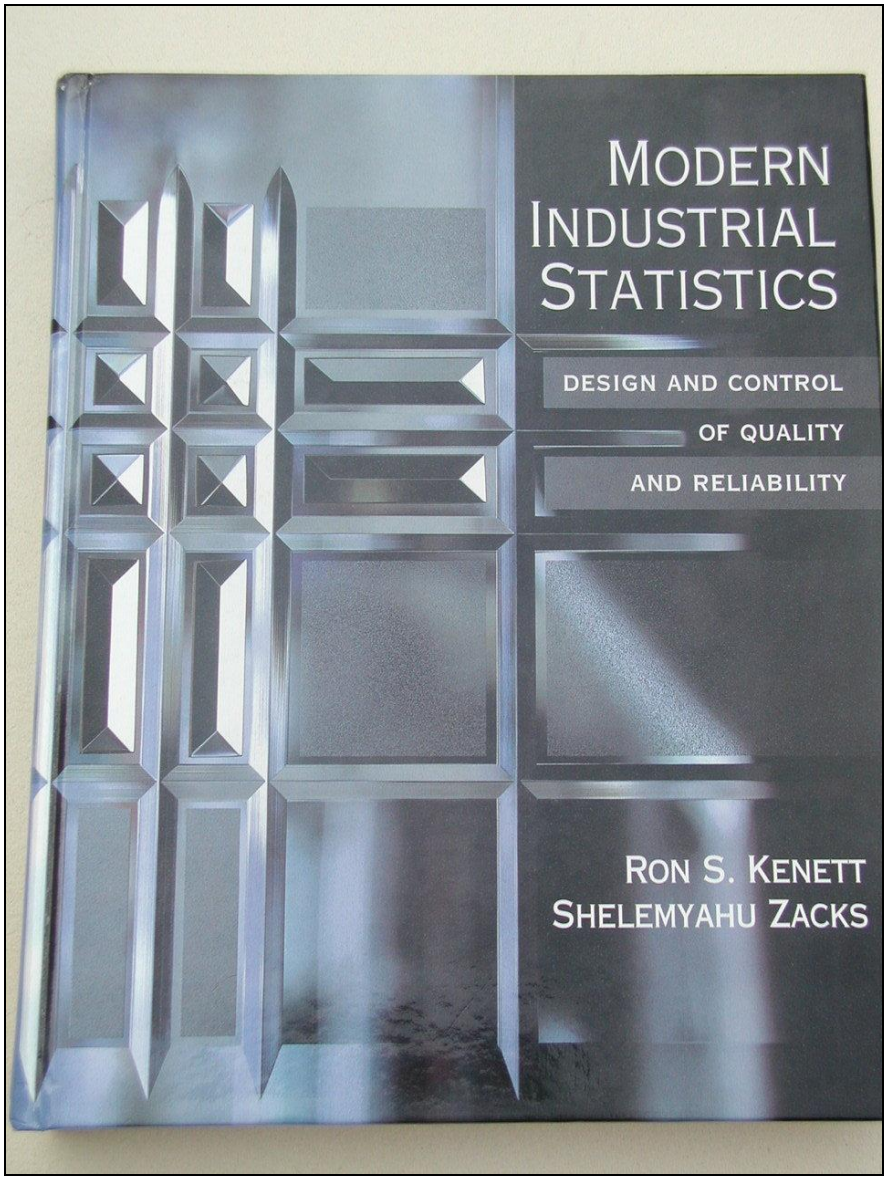
# Summary

- **Visualizing Multivariate Data**
  - » Scatter plots
  - » Bubble plots
- **Multivariate Process Control**
  - » $T^2$ charts
  - » Two examples
- **Multivariate Data Analysis**
  - » Association rules
  - » The Italian case study

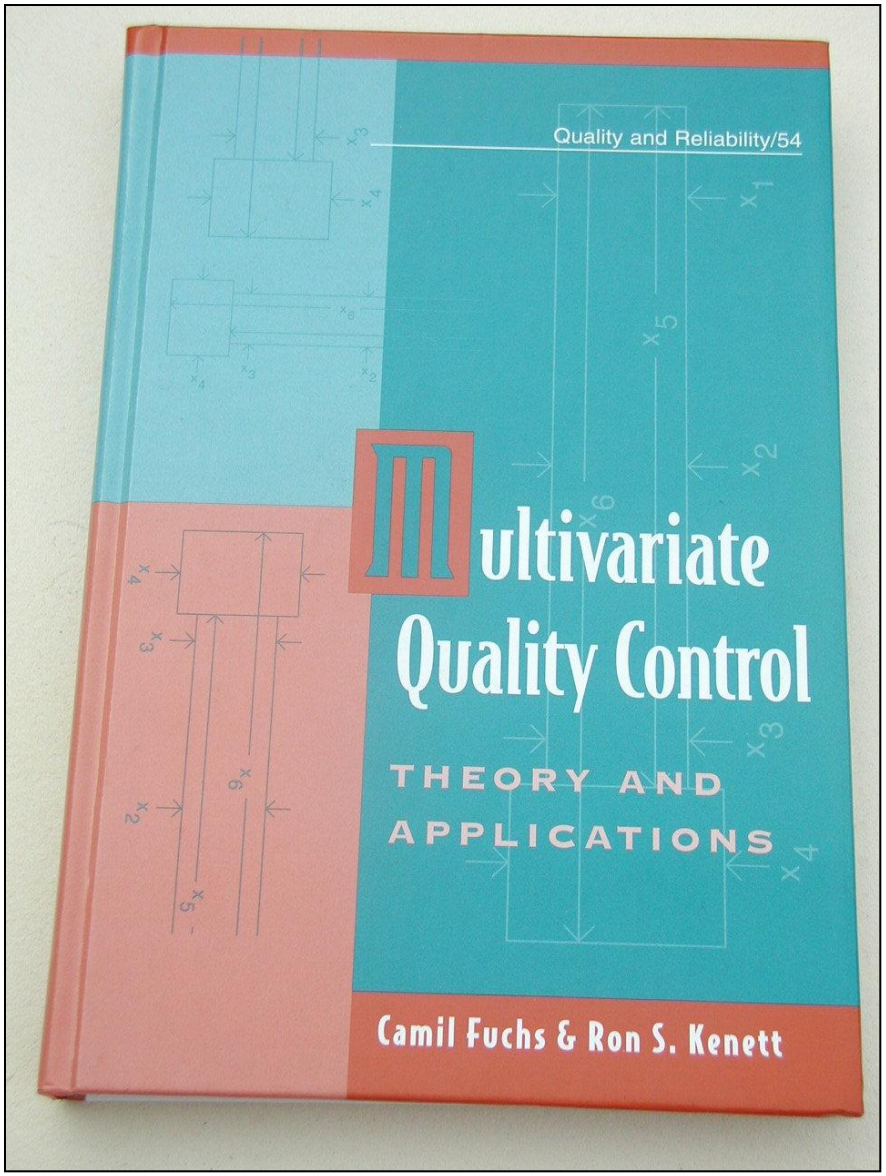


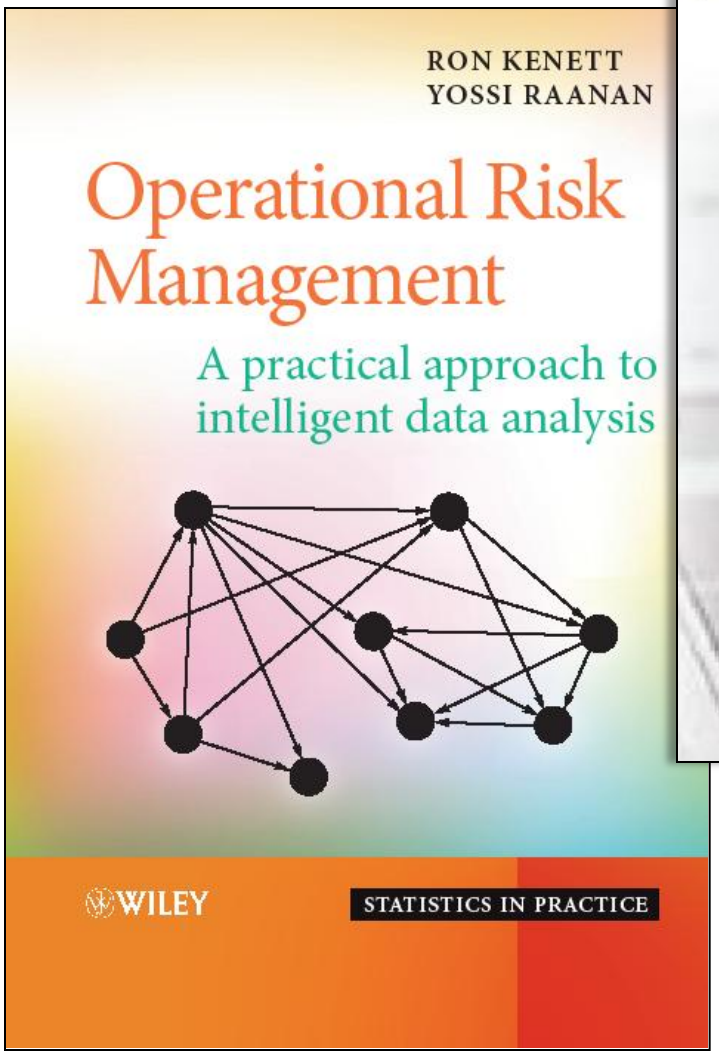






Kenett and Zacks, **Modern Industrial Control:** ***Design and control of quality and reliability***, Duxbury press: San Francisco, 1998




Fuchs and Kenett, **Multivariate Quality Control:** ***Theory and Applications***, M. Dekker: New York, 1998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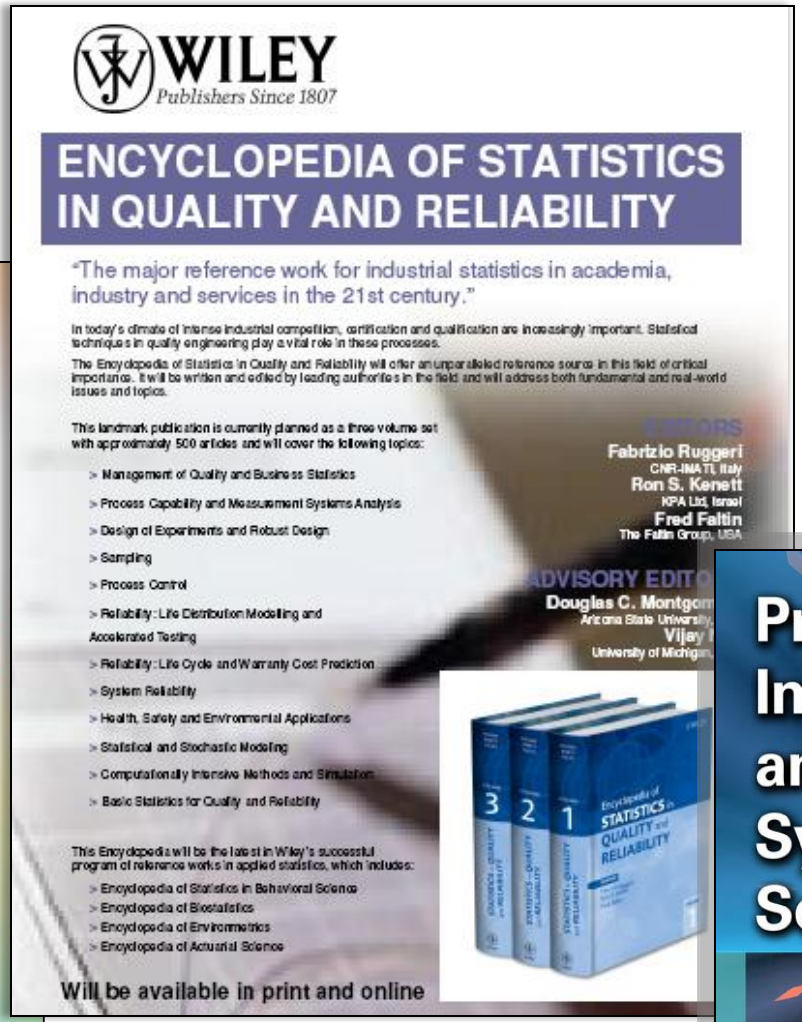
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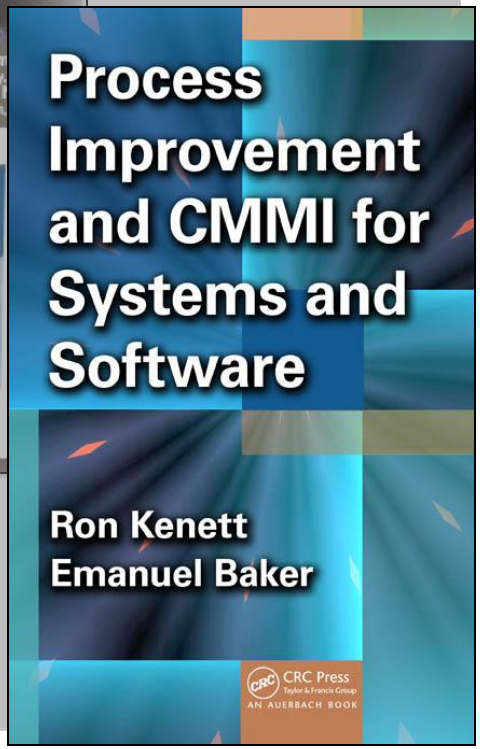
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